

WARM UP

Given that $x=37$, find the other variable.

$$x = y \quad \boxed{y=37} \quad x + t = 180 \quad 180 - 2x = m$$

$$37 + t = 180 \quad 140 - 74 = m$$

$$\boxed{t=143} \quad \boxed{106 = m}$$

If $r = 36$, $s = 34$, and $2r + 3s - x = n$, find n .

$$2(36) + 3(34) - 37 = n \quad \rightarrow \quad 174 - 37 = n$$

$$72 + 102 - 37 = n \quad \rightarrow \quad \boxed{137 = n}$$

ESSENTIAL QUESTION

What angle relationships are created when parallel lines are intersected by a transversal?

NEEDED VOCAB:

- ▶ Parallel Lines
- ▶ Transversals
- ▶ Corresponding Angles
- ▶ Alternate Interior Angles
- ▶ Alternate Exterior Angles
- ▶ Same-Side Interior Angles

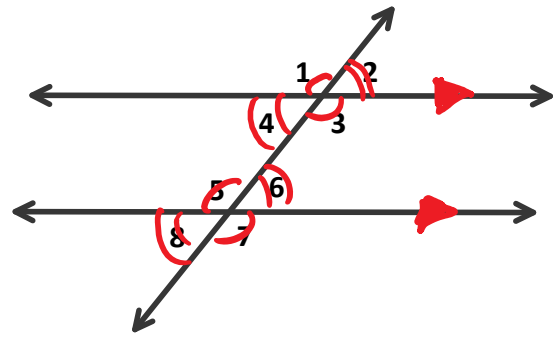
GOAL: "I CAN..."

Determine the measures of the angles formed when parallel lines are intersected by a transversal."

EXPLORE AND REASON

The diagram shows two parallel lines cut by a transversal.

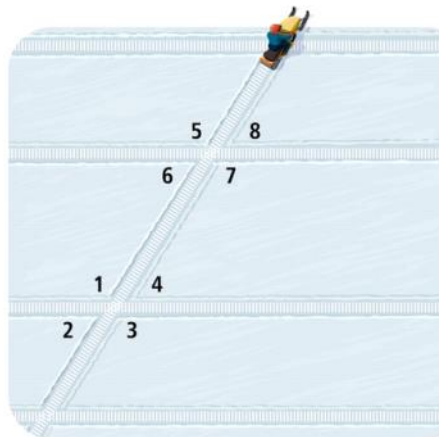
- A. What relationships do we already know the angles have with the three immediately around them?
- B. What relationships can we see that the angles will have to the angles of the other intersection?



EXAMPLE 1

Identify the pairs of angles of each angle type made by the snowmobile tracks.

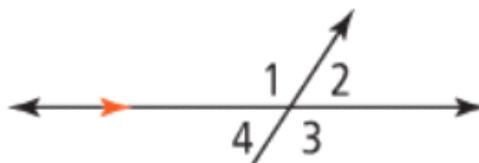
Alt. Ext:
 5, 3
 2, 8
Alt. Int:
 6, 4
 1, 7
SSI
 1, 6
 4, 7



vert. ∠'s:
 5, 7
 6, 8
 1, 3
 2, 4
Corresponding:
 1, 5
 2, 6
 3, 7
 4, 8

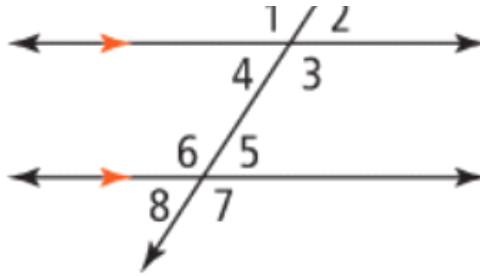
1. Which angle pairs include the named angle?

- a. $\angle 4$ *vert. : 4, 2*
 corr: 4, 8
 Alt. Int: 4, 5
- b. $\angle 5$ *vert: 5, 7*



b. $\angle 5$

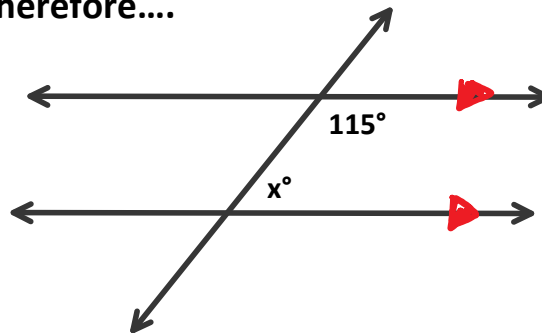
Alt. Int: 4, 5
vert: 5, 8
corr: 5, 2
Alt. Int: 5, 4



Same-Side Interior Angles Postulate

If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

Therefore....

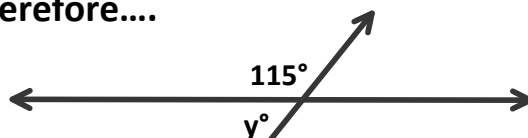


$$\begin{aligned}x + 115 &= 180 \\x &= 180 - 115 \\x &= 65\end{aligned}$$

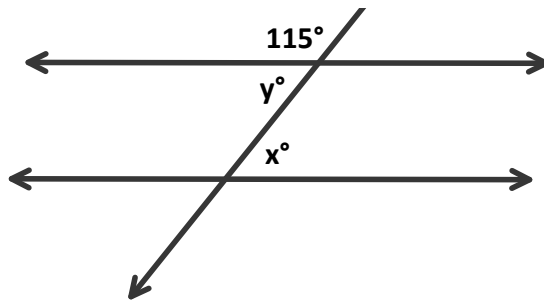
Alternate Interior Angles Theorem

If a transversal intersects two parallel lines, then alternate interior angles are congruent.

Therefore....



$$\begin{aligned}y + 115 &= 180 \\y &= 180 - 115 \\y &= 65\end{aligned}$$

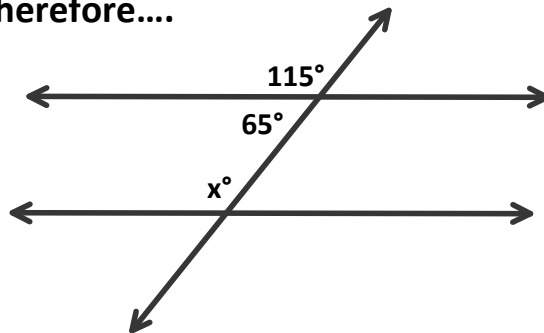


$$\begin{aligned}
 y + 115 &= 180 \\
 y &= 180 - 115 \\
 y &= 65 \\
 y &= x \\
 x &= 65
 \end{aligned}$$

Corresponding Angles Theorem

If a transversal intersects two parallel lines, then corresponding angles are congruent.

Therefore....

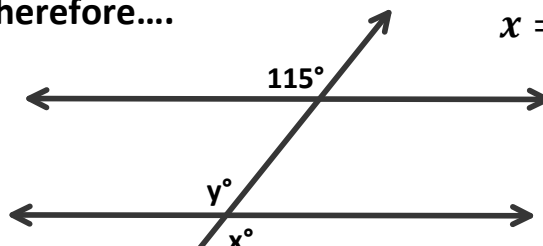


$$\begin{aligned}
 x + 65 &= 180 \\
 x &= 180 - 65 \\
 x &= 115
 \end{aligned}$$

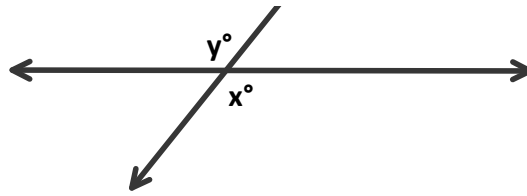
Alternate Exterior Angles Theorem

If a transversal intersects two parallel lines, then alternate exterior angles are congruent.

Therefore....



$$\begin{aligned}
 x &= y \\
 y &= 115 \text{ Corresponding Angles} \\
 x &= 115 \text{ Vertical Angles}
 \end{aligned}$$

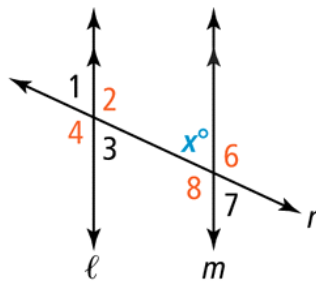


CHECKING FOR KNOWLEDGE

How do each of the angles relate to x° ?

x and 8 are sup.
 " " 6 " "
 " " 2 " "
 " " 1 are " "
 " " 7 " "
 " " 3 " "

$m\angle x + m\angle 2 = 180$
 $m\angle 2 = m\angle 4$
 $m\angle x + m\angle 4 = 180$

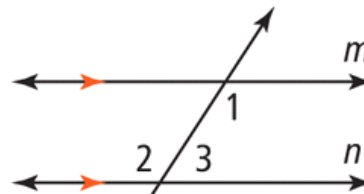


EXAMPLE 2

Prove the Alternate Interior Angles Theorem

Given: $m \parallel n$

Prove: $\angle 1 \cong \angle 2$



$m \parallel n$
 Given

$\angle 1$ & $\angle 3$ are supplementary
 same-side interior

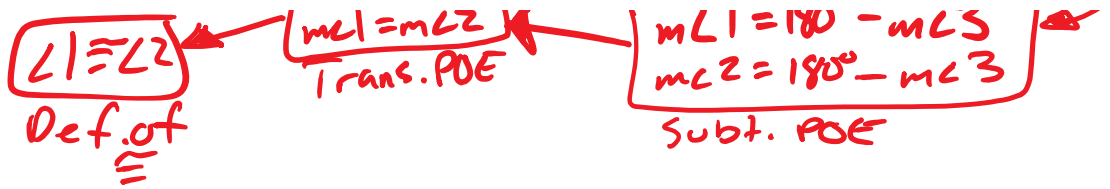
$m\angle 1 + m\angle 3 = 180^\circ$
 prop. of same side int.

$m\angle 2 + m\angle 3 = 180^\circ$
 Linear Pair

$m\angle 1 = 180^\circ - m\angle 3$
 $m\angle 2 = 180^\circ - m\angle 3$
 Subst. POE

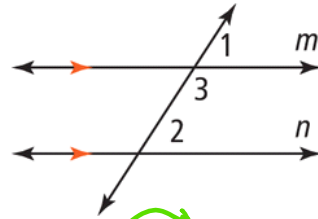
$m\angle 1 = m\angle 2$
 Trans. POE

$\angle 1 \cong \angle 2$
 Def. of \cong



3. Prove the Corresponding Angles Theorem.

Given: $m \parallel n$
 Prove: $\angle 1 \cong \angle 2$



$m \parallel n$, $\angle 1 + \angle 3$ are linear pair,
 $\angle 3$ and $\angle 2$ are SSI.
 Given

$m\angle 1 + m\angle 3 = 180^\circ$
 $m\angle 2 + m\angle 3 = 180^\circ$
 Prop. of Linear pair
 + SSI

$m\angle 1 = 180^\circ - m\angle 3$
 $m\angle 2 = 180^\circ - m\angle 3$
 Subst. POE

$m\angle 1 = m\angle 2$
 Trans. POE

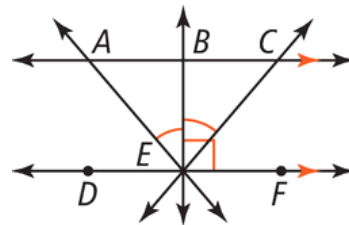
$\angle 1 \cong \angle 2$
 Def. of \cong

EXAMPLE 3

Use the diagram to prove the angle relationship.

Given: $\overline{AC} \parallel \overline{DF}$, $\overline{BE} \perp \overline{DF}$, and $\angle AEB \cong \angle CEB$

Prove: $\angle BAE \cong \angle BCE$



$\overline{AC} \parallel \overline{DF}$, $\overline{BE} \perp \overline{DF}$, $\angle AEB \cong \angle CEB$
 Given

$m\angle BEF = 90^\circ$
 Def. of \perp

$m\angle CBE + m\angle BEF = 180^\circ$
 Prop. of SSI

$m\angle CBE + 90^\circ = 180^\circ$
 Subs. POE

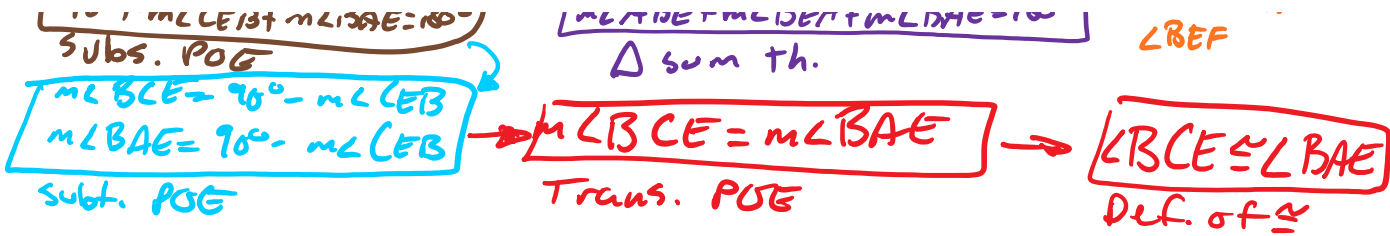
$m\angle CBE = 90^\circ$
 Subst. POE

$m\angle ABE = 90^\circ$
 Alt. Int. to $\angle BEF$

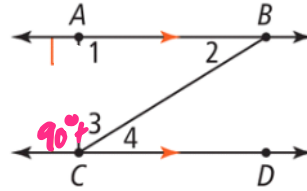
$90^\circ + m\angle CBE + m\angle BCE = 180^\circ$
 $90^\circ + m\angle CEB + m\angle BAE = 180^\circ$
 Subs. POE

$m\angle CBE + m\angle BEC + m\angle BCE = 180^\circ$
 $m\angle ABE + m\angle BEA + m\angle BAE = 180^\circ$
 Δ sum th.

$m\angle BCE = 90^\circ - m\angle CEB$



4. Given $\overline{AB} \parallel \overline{CD}$, prove that $m\angle 1 + m\angle 2 + m\angle 3 = 180$.



$\overline{AB} \parallel \overline{CD}, \overline{AC} \perp \overline{AB}, \overline{AC} \perp \overline{CD}$
 Given \downarrow $\angle 2 + \angle 3$ are SSI

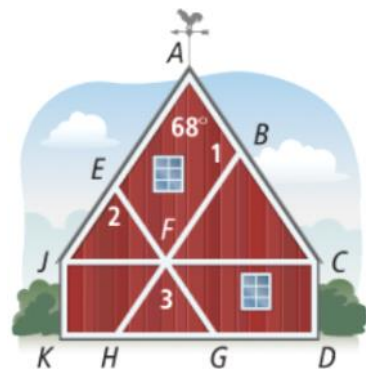
$m\angle 1 = 90^\circ$ \rightarrow $m\angle 2 + m\angle 3 + 90^\circ = 180^\circ$
 Def. of \perp Prop. of SSI

$m\angle 2 + m\angle 3 + m\angle 1 = 180^\circ$
 Subs. POE

EXAMPLE 4

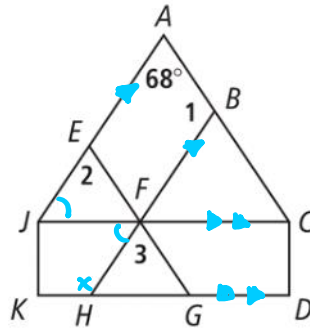
The white trim shown for the wall of a barn should be constructed so that $\overline{AC} \parallel \overline{EG}$, $\overline{JA} \parallel \overline{HB}$, and $\overline{JC} \parallel \overline{KG}$. What should $m\angle 1$ and $m\angle 3$ be?

$1 + 68 = 180$ $1 + F = 180$
 $m\angle 1 = 112^\circ$ $F = 3$
 $m\angle 3 = 68^\circ$ $1 + 3 = 180$



5. If $m\angle EJF = 56$, find $m\angle FHK$.

124°



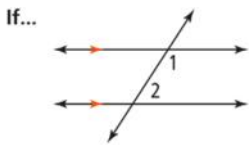
$$56 + x = 180$$

$$x = \underline{\underline{124}}$$

Parallel Lines and Angle Pairs

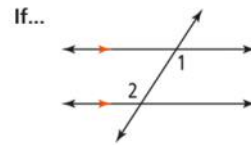
There are four special angle relationships formed when parallel lines are intersected by a transversal.

POSTULATE 2-1 Same-Side Interior Angles Postulate



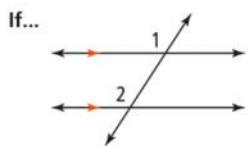
Then... $m\angle 1 + m\angle 2 = 180^\circ$

THEOREM 2-1 Alternate Interior Angles Theorem



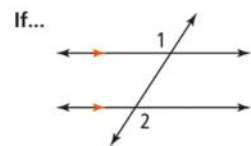
Then... $\angle 1 \cong \angle 2$

THEOREM 2-2 Corresponding Angles Theorem



Then... $\angle 1 \cong \angle 2$

THEOREM 2-3 Alternate Exterior Angles Theorem



Then... $\angle 1 \cong \angle 2$

HOMWORK

Pg. 76

10-13, 15-17, 19-23 ODD, 24, 28
