

3.4 Arithmetic Sequences

Monday, September 23, 2019 7:46 AM

WARM UP

Find the pattern within the number set.

2, 4, 6, 8, 10

1, 4, 7, 10, 13

12, 24, 36, 48, 60

12, 16, 20, 24, 28

13, 26, 39, 52, 65

2.5, 4, 5.5, 7, 8.5

ESSENTIAL QUESTION

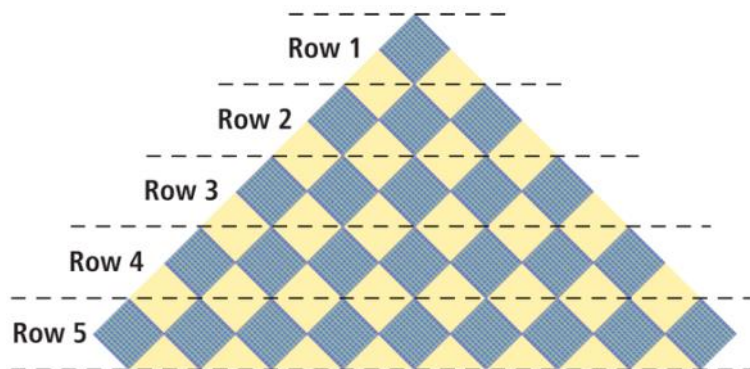
How are arithmetic sequences related to linear functions?

NEEDED VOCAB:

- ▶ **Arithmetic Sequence**
- ▶ **Common Difference**
- ▶ **Explicit Formula**
- ▶ **Recursive Formula**
- ▶ **Sequence**
- ▶ **Term of the Sequence**

GOAL: "I CAN. . .

Identify and describe arithmetic sequences."



There are 5 shaded squares in row 3. The total number of shaded squares up to and including row 3 is 9. Fill in the table for the remaining rows.

Row Number	1	2	3	4	5
# of Shaded Squares in the Row	1		5		
Total # of Shaded Squares	1		9		

A **sequence** is an ordered list of numbers that often forms a pattern. Each number is a **term of the sequence**. In an **arithmetic sequence**, the difference between any two consecutive terms is a constant called the **common difference**.

EXAMPLE 1

Is the ordered list 26, 39, 52, 65, 78 an arithmetic sequence?

Sequences relate to functions where the number of the term is our x and the value of the term is our y. So that means our domain relates to the term number and our range relates to the term value.

The arithmetic sequence form before was 26, 39, 52, 65, 78. So that means if the function of the sequence is A:

Function Notation

$$A(1)=26$$

The 1 term is 26.

$$A(2)=39$$

The 2 term is 39.

$$A(3)=52$$

The 3 term is 52.

and so on...

Subscript Notation

$$A_1 = 26$$

$$A_2 = 39$$

$$A_3 = 52$$

and so on...

If $A(n)$ is the nth term of the sequence, 26, 39, 52, 65, 78.

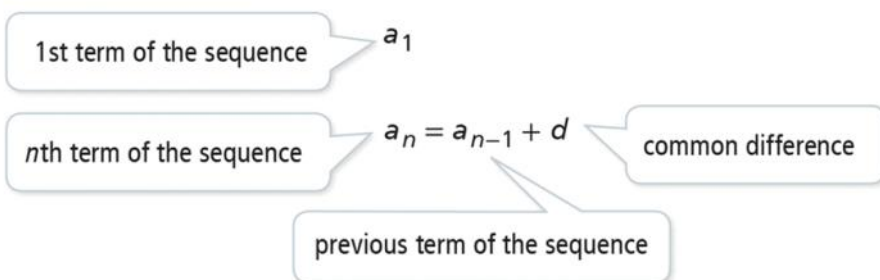
Is the Domain of the sequence discrete or continuous?

Explain.

Recursive Definition

Recursive, in mathematics, means to repeat a process over and over again, using the output of each step as the next input. A recursive formula relates each term of a sequence to the previous term. It is composed of an initial value and a rule for generating the sequence.

The **recursive formula** for an arithmetic sequence is:



A recursive formula describes the pattern of a sequence and can be used to find the next term in a sequence.

EXAMPLE 2

What is a recursive formula for the height above the ground of the n th step of the pyramid shown?



Using the formula we just found, $A_n = A_{n-1} + 26$, where $A_1 = 26$, let's find the height off the ground of the 3rd step.

Rewrite the recursive formula if the height of each step is changed from 26 to 18.

Explicit Formula

An **explicit formula** expresses the n th term of a sequence in terms of n .

The explicit formula for an arithmetic sequence is:

$$a_n = a_1 + (n - 1)d$$

Diagram illustrating the components of the explicit formula for an arithmetic sequence:

- a_1 : first term of the sequence
- n : term number
- d : common difference

EXAMPLE 3

The cost of renting a bicycle is given in the table below. How can you represent the rental cost using an explicit formula?

# of days rented	1	2	3	4
Rental cost	26	38	50	62

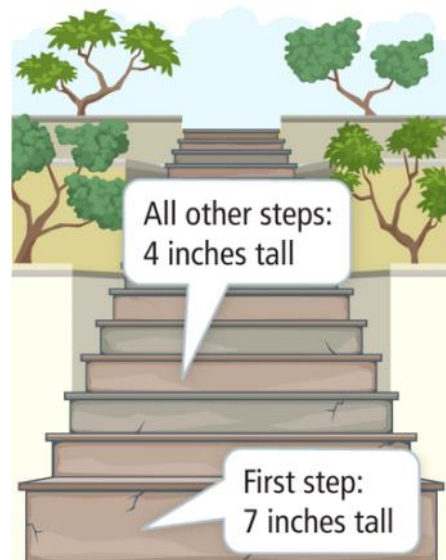
What is the cost of renting the bicycle for 10 days? The explicit formula is $a_n = 14 + 12n$ gives the rental cost for n days.

How is the explicit formula of an arithmetic sequence related to a linear function?

The cost to rent a bike is \$28 for the first day plus \$2 for each day after that. Write an explicit formula for the rental cost for n days. What is the cost to rent the bike for 8 days?

EXAMPLE 4

The recursive formula for the height above the ground of the n th step of the stairs shown is $a_n = a_{n-1} + 4$ with $a_1 = 7$. What explicit formula finds the height above the ground of the n th step?



Write an explicit formula for each arithmetic sequence.

$$a_n = a_{n-1} - 3; a_1 = 10$$

$$a_n = a_{n-1} + 2.4; a_1 = -1$$

EXAMPLE 5

The explicit formula for an arithmetic sequence is $a_n = 1 + \frac{1}{2}n$. What is the recursive formula for the sequence?

Write a recursive formula for each explicit formula.

$$a_n = 8 + 3n$$

$$a_n = 12 - 5n$$

Arithmetic Sequences

WORDS An arithmetic sequence is a sequence of numbers that follows a pattern. The difference between two consecutive terms is a constant called the common difference.

FORMULAS

Recursive Formula

Used to describe a sequence and find the next few terms

$$a_n = a_{n-1} + d$$

n th term of the sequence

previous term of the sequence

common difference

Explicit Formula

Used to find a specific term in the sequence

$$a_n = a_1 + (n - 1)d$$

n th term of the sequence

first term of the sequence

common difference

The first term of the sequence is a_1 .

NUMBERS

1, 7, 13, 19, 25, ...

Use the recursive formula to describe the sequence and find the next two terms.

$$a_n = a_{n-1} + 6$$

The first term is 1.

$a_1 = 1$
 $a_2 = 7$
 $a_3 = 13$
 $a_4 = 19$
 $a_5 = 25$

The common difference is 6.

$$\begin{aligned} a_6 &= a_5 + 6 \\ &= 25 + 6 \\ &= 31 \end{aligned}$$

$$\begin{aligned} a_7 &= a_6 + 6 \\ &= 31 + 6 \\ &= 37 \end{aligned}$$

The next two terms are 31 and 37.

Use the explicit formula to find the 15th term in the sequence.

$$a_n = 1 + (n - 1)6$$

$$a_{15} = 1 + (14)6$$

$$a_{15} = 85$$

HOMEWORK

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14, 17-28, 32-36 EVEN, 45, 47