

## WARM UP

Simplify each expression.

$$\frac{y^4 x^2}{y^4 x^3}$$

$$\frac{x^2}{x^3} = \boxed{\frac{1}{x}}$$

$$\frac{2^3 c^1 x^1}{2^9 c^5 x^4}$$

$$\frac{2^3}{c^5 x^4} = \boxed{\frac{8}{c^5 x^4}}$$

$$\frac{9^0 y^2 x^4}{9^2 y^7 x^8}$$

$$\frac{1 \cdot y^2 x^4}{9^2 \cdot 1 \cdot 1} = \boxed{\frac{y^2 x^4}{81}}$$

## ESSENTIAL QUESTION

What are the properties of rational exponents and how are they used to solve problems?

**NEEDED VOCAB:**

► **Rational Exponents**

**GOAL: "I CAN..."**

**Use properties of exponents to solve equations with rational exponents."**

Students are asked to write an equivalent expression for  $3^{-3}$ .

Casey and Jacinta each write an expression on the board.

Casey  
 $3^{-3} = -27$

Jacinta  
 $3^{-3} = \frac{1}{27}$

• negative powers means to flip the fraction. ex:  $2^{-1} = \frac{1}{2}$ ,  
 $2^{-2} = \frac{1}{4}$ .

$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

Which, if any, are correct?

Jacinta is correct.

### EXAMPLE 1

What does  $9^{\frac{1}{2}}$  equal?

$9^{\frac{1}{2}} = ? = x$

$9^{\frac{1}{2}} = x$

$(9^{\frac{1}{2}})^2 = x^2 = 9^1$

$x = \sqrt{9} = 3$

Power to a power (multiply)  
 $\frac{1}{2} \cdot 2 = 1$

we square root to find answer.

which means fraction powers are roots.

$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

or  $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$   
or  $27^{\frac{2}{3}} = \sqrt[3]{27^2} = \sqrt[3]{729} = 9$

What does  $8^{\frac{1}{3}}$  equal?

$8^{\frac{1}{3}} = \sqrt[3]{8} = \boxed{2}$

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**EXAMPLE 2** Solve for the value of  $x$ .

$$2^x \cdot 2^{2x} = 2^6$$

Same base means add exponents.

$$2^x \cdot 2^{2x} = 2^6$$

$$2^{3x} = 2^6$$

Since the bases are =  
the powers must be =.

$$\begin{array}{l} 3x = 6 \\ \hline x = 2 \end{array}$$

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What is the solution of  $(3^{\frac{x}{2}})(3^{\frac{x}{3}}) = 3^9$ ?

same as before but now  
using fractions.

$$3^{(\frac{x}{2} + \frac{x}{3})} = 3^9$$

$$\left(\frac{3}{3}\right)\frac{x}{2} + \left(\frac{2}{2}\right)\frac{x}{3} = 9$$

$$\frac{3x}{6} + \frac{2x}{6} = 9$$

$$\frac{5x}{6} = 9$$

$$5x = 54$$

$$\boxed{x = 10.8}$$

What is the solution of  $(2^{\frac{x}{4}})(2^{\frac{x}{6}}) = 2^3$ ?

same as before

$$2^{\frac{x}{4}} \cdot 2^{\frac{x}{6}} = 2^3$$

$$\left(\frac{3}{3}\right)\frac{x}{4} + \left(\frac{2}{2}\right)\frac{x}{6} = 3$$

$$\frac{3x}{12} + \frac{2x}{12} = 3$$

$$\frac{5x}{12} = 3$$

$$5x = 36$$

$$\boxed{x = 7.2}$$

**EXAMPLE 3** What is the solution of  $27^{x-4} = 3^{2x-6}$ ?

- make them have same base.
- $3^3 = 27$ , so substitute.

$$(3^3)^{x-4} = 3^{2x-6}$$

$$3^{3x-12} = 3^{2x-6} \quad \leftarrow \text{Power to Power}$$

$$3x - 12 = 2x - 6$$

$$\boxed{x = 6}$$

Bases are equal so to must be the powers.

$$27^{6-4} = 3^{2(6)-6} \quad \checkmark \checkmark$$

$$27^2 = 3^{12-6}$$

$$729 = 3^6$$

$$729 = 729 \quad \checkmark$$

What is the solution of  $\left(\frac{1}{125}\right)^{-\frac{x}{2}} = \left(\frac{1}{25}\right)^{-\frac{x}{3}-2}$ ?

need to convert both to same base

$$\frac{1}{125} = 5^{-3} \quad \frac{1}{25} = 5^{-2}$$

$$\left(5^{-3}\right)^{-\frac{x}{2}} = \left(5^{-2}\right)^{-\frac{x}{3}-2}$$

now use power to power rule

$$5^{\frac{3x}{2}} = 5^{\frac{2x}{3}+4}$$

bases are = so too must be the powers.

$$\frac{3x}{2} = \frac{2x}{3} + 4$$

What is the solution of each equation?

a.  $256^{x+2} = 4^{3x+9}$

$$(4^4)^{x+2} = 4^{3x+9}$$

$$4^{4x+8} = 4^{3x+9}$$

$$4x+8 = 3x+9$$

$$\boxed{x=1}$$

b.  $\left(\frac{1}{8}\right)^{\frac{x}{2}-1} = \left(\frac{1}{4}\right)^{\frac{x}{3}}$

$$(2^{-3})^{\frac{x}{2}-1} = (2^{-2})^{\frac{x}{3}}$$

$$-\frac{3x}{2}+3 = -\frac{2x}{3}$$

$$-\frac{9x}{2}+9 = -2x$$

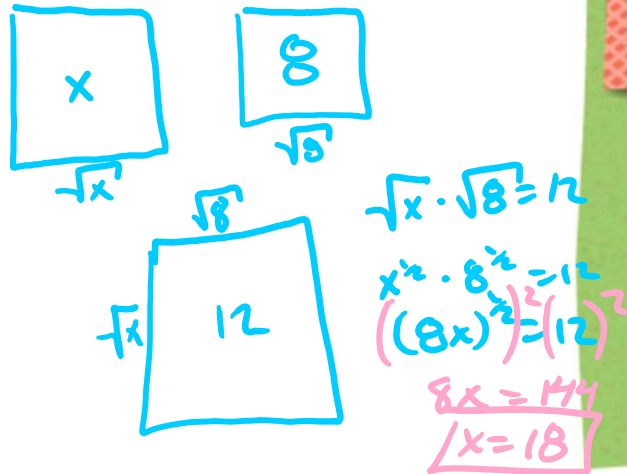
$$-9x+18 = -4x$$

$$18 = 5x$$

$$\boxed{3.6 = x}$$

### EXAMPLE 4

Adam is setting up for an outdoor concert. He places three square blankets near the band as shown in the picture. What is the area of Blanket C?



When the side length of Blanket A is multiplied by  $2^{\frac{1}{2}}$ , the result is 6 yards. Find the area of Blanket A.

$$t^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} = 6$$

$$(2t)^{\frac{1}{2}} = 6$$

$$2t = 36$$

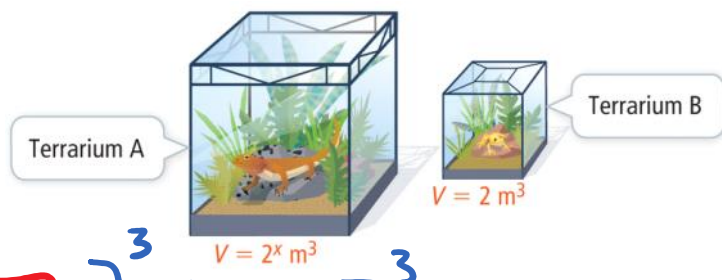
$$t = 18$$

$t$  →  
area




### EXAMPLE 5

Terrarium A and Terrarium B are cubes. The side length of Terrarium A is twice the side length of Terrarium B. What is



Terrarium A is twice the side length of Terrarium B. What is the value of  $x$ ?



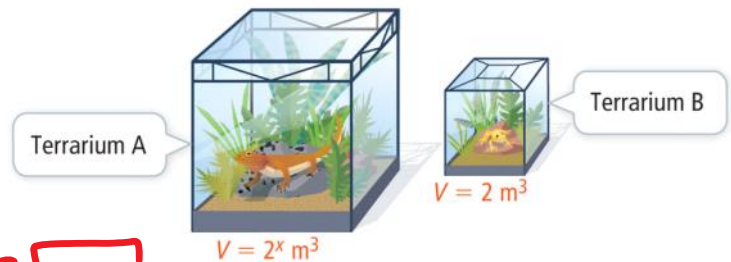
$$\left[ \frac{\sqrt[3]{2^x}}{2} \right]^3 = \left[ \sqrt[3]{2} \right]^3$$

$$\frac{2^x}{8} = 2$$

$$2^x = 16$$

$$2^x = 2^4 \quad \boxed{x=4}$$

What is the value of  $x$  if the side length of Terrarium A is **FOUR** times the length of Terrarium B?



$$\sqrt[3]{2^x} = 4 \cdot \sqrt[3]{2}$$

$$2^x = 4^3 \cdot 2^1$$

$$2^x = 2^6 \cdot 2^1$$

$$\boxed{x=7}$$

<https://tinyurl.com/uvpaf4r>



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# HOMWORK

**Pg. 222**

**21, 25-30, 31-42, 47, 49**