7.1 Dilations

Monday, September 23, 2019 7:46 AM

Warm Up

WITH A BUDDY

(or someone sitting next to you)

Draw ΔABC with points A(3,0), B(0,0), C(0,3).

Draw ΔA'B'C' with points A'(6,0), B'(0,0), C'(0,6)

What is the measure of the side lengths:

$\overline{AB} = 3$ $\overline{A'B'} = 6$

$\overline{BC} = 3$ $\overline{B'C'} = 6$

$\overline{AC} = 3\sqrt{2}$ $\overline{A'C'} = 6\sqrt{2}$

Essential Question

How does a dilation affect the side lengths and angle measures of a figure?

Needed Vocab:

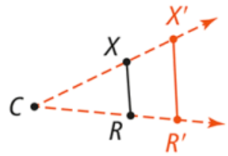
- **Center Dilation**

Goal: "I can...
Dilate figures and identify characteristics of dilation."

Dilations

A dilation $D_{(n,\ C)}$ is a transformation that has center of dilation C and scale factor n, where $n > 0$, with the following properties:

- Point R maps to R' in such a way that R' is on $\overrightarrow{CR}$ and $CR' = n \cdot CR$.
- Each length in the image is n times the corresponding length in the preimage (i.e., $X'R' = n \cdot XR$).

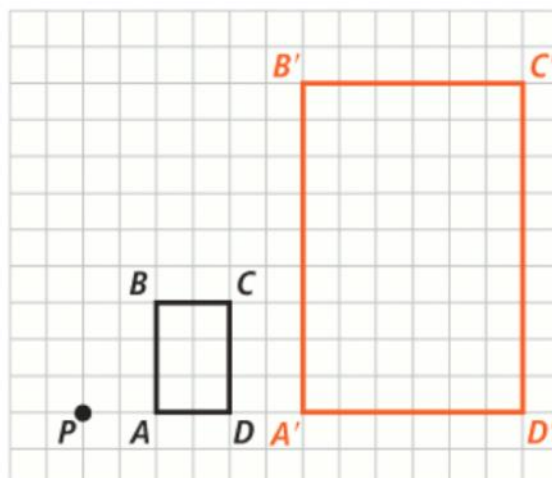


- The image of the center of dilation is the center itself (i.e., $C' = C$).
- If $n > 1$, the dilation is an *enlargement*.
- If $0 < n < 1$, the dilation is a *reduction*.
- Every angle is congruent to its image under the dilation.

On a coordinate plane, the notation D_n describes the dilation with the origin as center of dilation.

Example 1

Rectangle $A'B'C'D'$ is a dilation with center P of $ABCD$. How are the side lengths and angle measures of $ABCD$ related to those of $A'B'C'D'$?



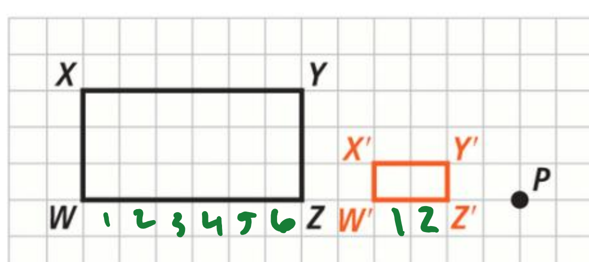
$AD = 2$
$A'D' = 6$
$\cdot 3$

$AB = 3$
$A'B' = 9$
$\cdot 3$

All angles are the same

Dilations change the lengths of sides or the size of the figure by a scale factor, K, while all angle measures are preserved.

2. Rectangle $W'X'Y'Z'$ is a dilation with center P of $WXYZ$. How are the side lengths and angle measures of the two figures related?



$K = ?$

$\boxed{K = 3}$

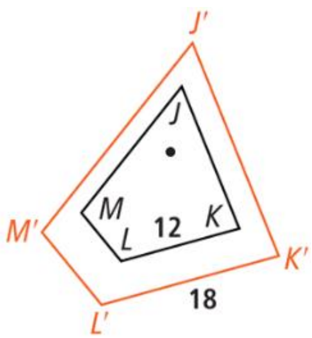
$\boxed{K=3}$

2:6
1:3

$\boxed{\text{All angle are the same}}$

Example 2

Quadrilateral $J'K'L'M'$ is a dilation of $JKLM$. What is the scale factor?



12:18
6:9
2:3

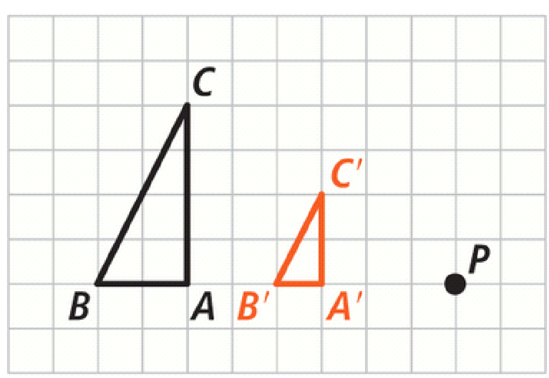
$\frac{3}{2}$ or 1.5

$\boxed{K = \frac{3}{2} \text{ or } 1.5}$

3. Consider the dilation shown.

a. Is the dilation an enlargement or a reduction?

b. What is the scale factor?



2:1

so

$\boxed{K = \frac{1}{2}}$

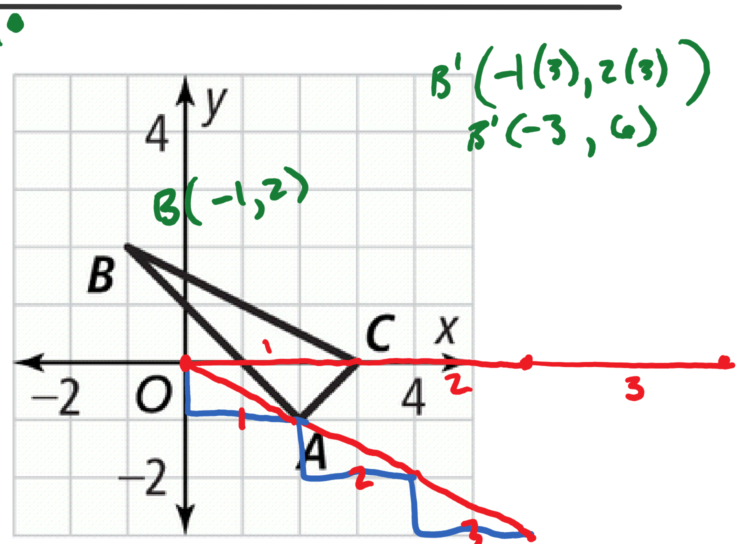
ΔABC is larger than ΔA'B'C' so

$\boxed{\text{Reduction}}$

EXAMPLE 3

What are the vertices of $D_3(\triangle ABC)$?

$B'(-1(3), 2(3))$
$B'(-3, 6)$



The notation $D_3(\triangle ABC)$ means the image of $\triangle ABC$ after a dilation centered at the origin, with scale factor 3.

By multiplying scale factor to the points themselves we can get the dilation point

$\boxed{A'(6,-3) \quad B'(-3,6) \quad C'(9,\emptyset)}$

By using distance from $\emptyset,\emptyset$ (center of dilation) and multiplying by scale factor we can find the dilation vertices.

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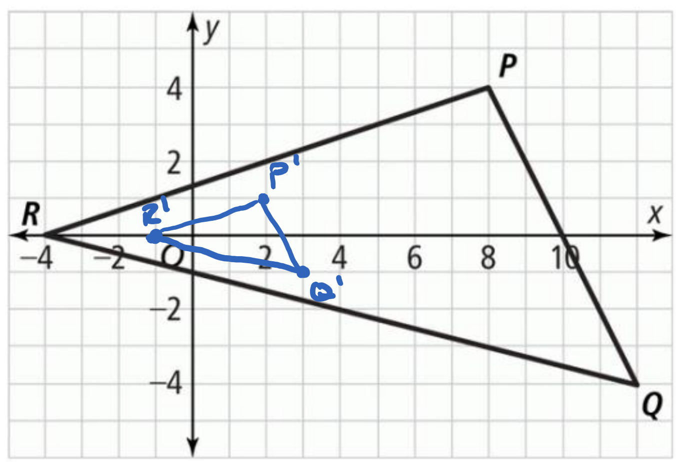
4. Use $\triangle PQR$.



a. What are the vertices of $D_{\frac{1}{4}}(\triangle PQR)$?

means ÷ 4

$P'(2,1)$, $Q'(3,-1)$, $R'(-1,\emptyset)$

b. How are the distances to the origin from each image point related to the distance to the origin from each corresponding

exactly 1/4 the original distance.

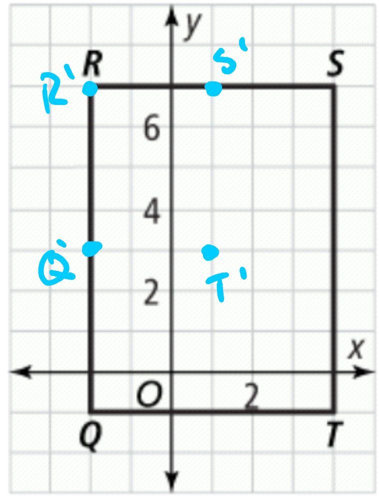
Example 4

What are the vertices of $D_{(\frac{1}{2},\ R)}(QRST)$?

Dilation

scale factor (K)

center of Dilation.



$RQ = 8$

$\therefore R'Q' = 4$

$QT = 6$

$\therefore Q'T' = 3$

$R = R'$

$\boxed{R'(-2,7)}$

$\boxed{Q'(-2,3)}$

$\boxed{T'(1,3)}$ $\boxed{S'(1,7)}$

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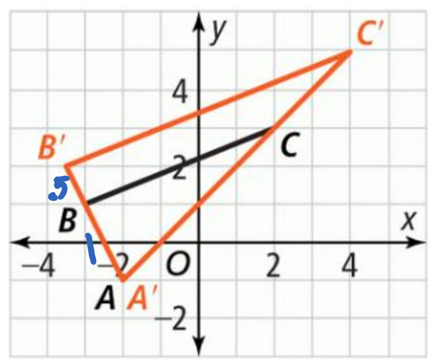
5. A dilation of $\triangle ABC$ is shown.



a. What is the center of dilation? $A = A'$ so $\boxed{\text{A is center of dilation}}$

b. What is the scale factor?

Enlargement so $K > 1$

$\boxed{K = 1.5 \text{ or } \frac{3}{2}}$

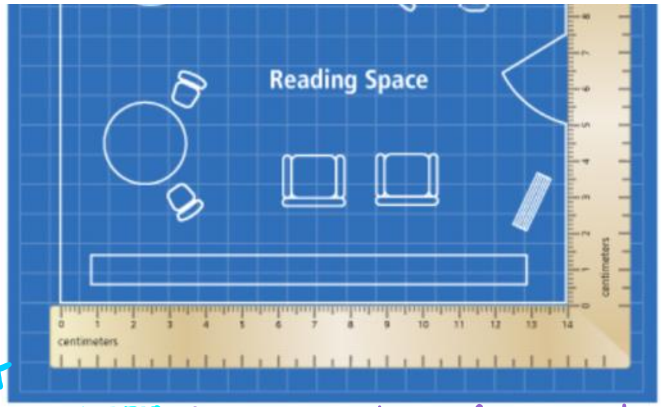
Either use logic and slopes or calculate distances w/distance formula.

Example 5

A blueprint for a new library uses a scale factor of $\frac{1}{50}$. Mr. Ayer measures the reading space on the blueprint to find the actual dimensions and area so he can order furniture.

1:50

area so he can order furniture. 1:50



A. What are the actual dimensions of the reading space?

B. What is the actual area of the reading space? How does the actual area relate to the area on the blueprint?

14 · 12 = 168 · 50 = 8400

not correct

14 x 12

14·50 x 12·50

$\boxed{700_{cm} \times 600_{cm}}$

700 · 600

168 · 2500

$\boxed{420000\ cm^2}$

Linear ratio 1:K 1:50

Area ratio 1:K² 1:2500

Volume ratio 1:K³ 1:125000

6. A blueprint for a house uses a scale factor of $\frac{1}{20}$. 1:20

a. If the dimensions of the actual kitchen are 3.1 m by 3.4 m, what are the dimensions of the kitchen on the blueprint?

3.1 x 3.4

$\frac{3.1}{20} \times \frac{3.4}{20} \rightarrow \boxed{0.155\,m \times 0.17\,m}$

b. What is the relationship between the area of the actual kitchen and the area of the kitchen on the blueprint?

L 1:20

$\boxed{\text{A } 1:400}$

Homework

Pg. 308
10, 13, 14, 18, 20, 22, 25, 27