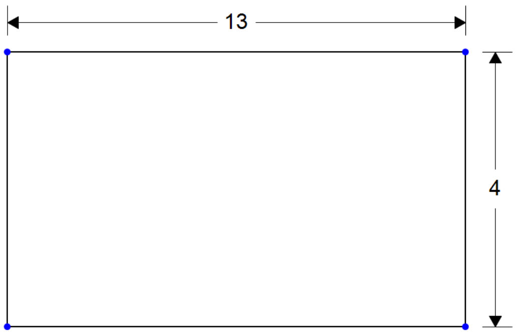
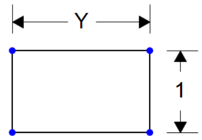
Warm Up

Find the missing measure to make the objects similar.



$\frac{y}{13} = \frac{1}{4}$

$\boxed{y = \frac{13}{4}}$

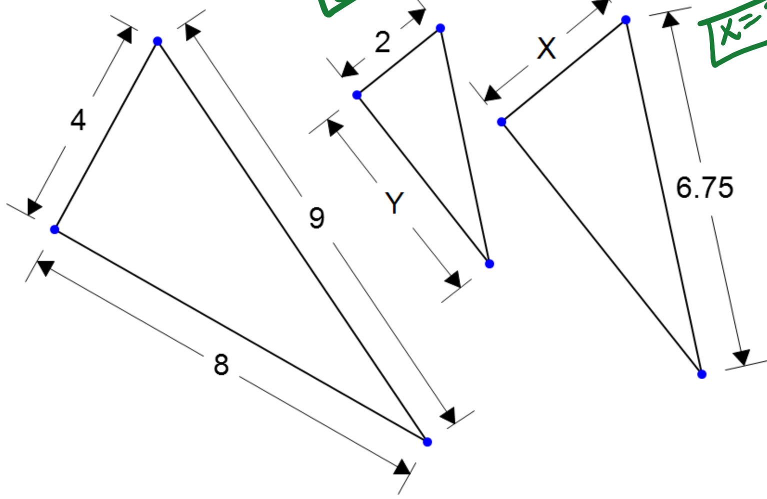


$\boxed{y=4}$

$\frac{x}{6.75} = \frac{4}{9}$

$x = \frac{27}{9}$

$\boxed{x=3}$



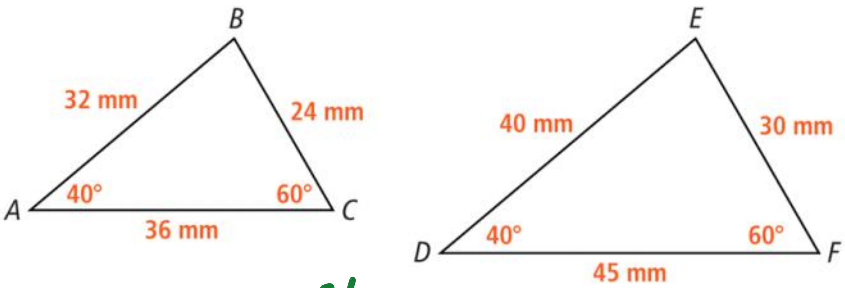
Essential Question

How can you use the angles and sides of two triangles to determine whether they are similar?

Goal: "I can. . .

Use dilation and rigid motion to establish triangle similarity theorems."

Do you think that the two triangles are similar?



$$\frac{32}{40} = \frac{24}{30} = \frac{36}{45}$$

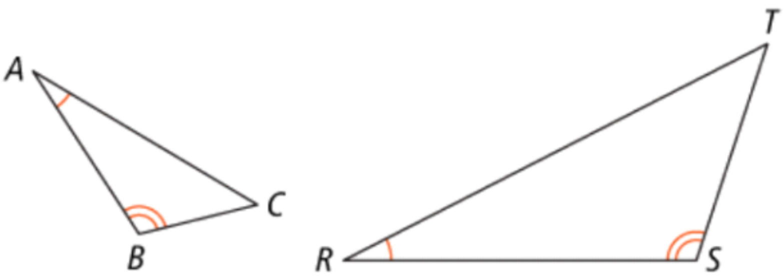
$$\downarrow \quad \downarrow \quad \downarrow$$

$$\frac{4}{5} \quad \frac{4}{5} \quad \frac{4}{5}$$ yes similar.

Would any triangle with two angles measuring 40∘ and 60∘ be similar to ΔABC? yes, third angle has to be 80°.

Example 1

If $\angle A \cong \angle R$ and $\angle B \cong \angle S$, is $\triangle ABC \sim \triangle RST$? Explain.



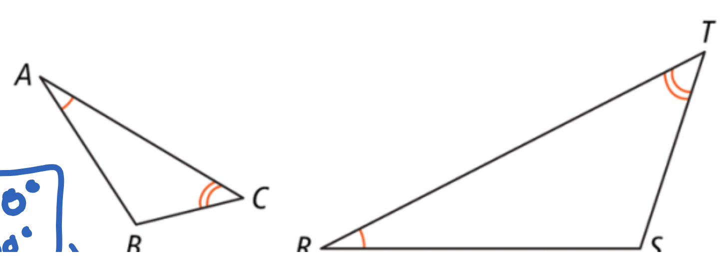
yes.

$m\angle A + m\angle B + m\angle C = 180°$
$m\angle R + m\angle S + m\angle T = 180°$

$m\angle A + m\angle B + m\angle T = 180°$
$m\angle A + m\angle B + m\angle C = m\angle A + m\angle B + m\angle T$
$m\angle C = m\angle T$ ✓

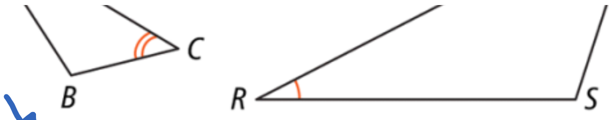
so if any two angles are ≅ the third must also be ≅.

1. If $\angle A$ is congruent to $\angle R$, and $\angle C$ is congruent to $\angle T$, how would you prove the triangles are similar?



$\angle A \cong \angle R$ → $m\angle A + m\angle C + m\angle B = 180°$

triangles are similar?



$\boxed{\begin{array}{l}\angle A \cong \angle R \\ \angle C \cong \angle T\end{array}}$
Given

$\boxed{\begin{array}{l}m\angle A + m\angle C + m\angle B = 180° \\ m\angle R + m\angle T + m\angle S = 180°\end{array}}$
Δ sum.

$\boxed{m\angle A + m\angle C + m\angle S = 180°}$
subs. POC

$\boxed{m\angle A + m\angle C + m\angle S = m\angle A + m\angle C + m\angle B}$
Trans. POE

$\boxed{m\angle S = m\angle B}$
Subt. POE

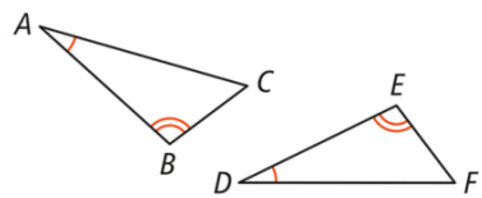
$\boxed{AAA}$
similarity th.

Angle-Angle Similarity (AA ~) Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

PROOF: SEE EXERCISE 10.

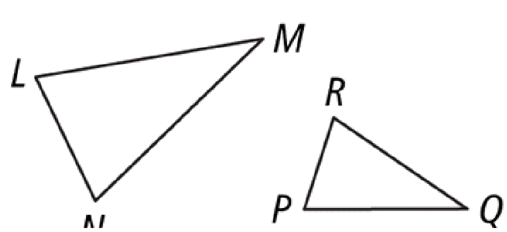
If...



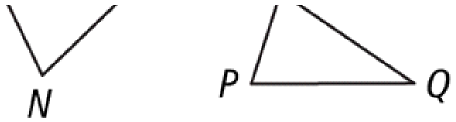
$\angle A \cong \angle D$ and $\angle B \cong \angle E$

Then... $\triangle ABC \sim \triangle DEF$

If $\dfrac{LM}{PQ} = \dfrac{MN}{QR} = \dfrac{LN}{PR}$, **is there a similarity transformation that maps $\triangle PQR$ to $\triangle LMN$? Explain.**



yes, translate Δ PQR so that R coincides with N. Rotate ΔPQR around R so that $\overline{RQ}$ coincides with $\overline{NM}$. Reflect ΔPQR Across $\overline{RQ}$. Dilate ΔPQR by scale factor k so that RQ•N= NM.



Across $\overline{RQ}$. Dilate $\triangle PQR$ by scale fac
k so that $RQ \cdot N = NM$.

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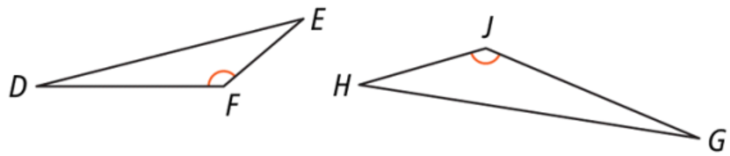
2. If $\dfrac{DF}{GJ} = \dfrac{EF}{HJ}$ and $\angle F \cong \angle J$, is there a similarity transformation that maps $\triangle DEF$ to $\triangle GHJ$? Explain.



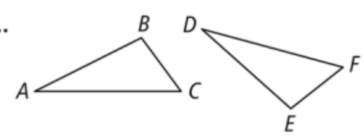
Translate $\triangle DEF$ so that point F coincides with point J. Rotate $\triangle DEF$ around point F so that $\overline{FE}$ coincides with $\overline{JH}$. Dilate $\triangle DEF$ by scale factor k so that $FE \cdot K = JH$.

Side-Side-Side Similarity (SSS ~) Theorem

If the corresponding sides of two triangles are proportional, then the triangles are similar.

PROOF: SEE EXERCISE 20.

If...



$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

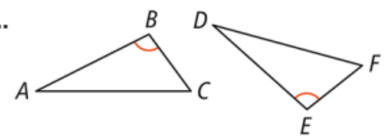
Then... $\triangle ABC \sim \triangle DEF$

Side-Angle-Side Similarity (SAS ~) Theorem

If an angle of one triangle is congruent to an angle of a second triangle, and the sides that include the two angles are proportional, then the triangles are similar.

PROOF: SEE EXERCISE 13.

If...

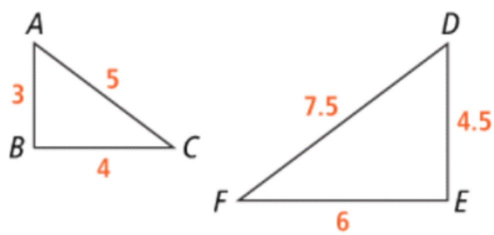


$\angle B \cong \angle E$ and $\frac{AB}{DE} = \frac{BC}{EF}$

Then... $\triangle ABC \sim \triangle DEF$

A. Are $\triangle ABC$ and $\triangle DEF$ similar?

No angles use SSS



$\frac{4.5}{3} = \frac{6}{4} = \frac{7.5}{5}$

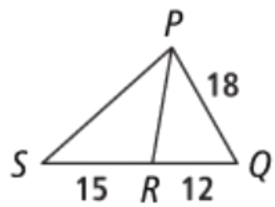
$\downarrow$

$\frac{1.5}{1} = \frac{1.5}{1} = \frac{1.5}{1}$

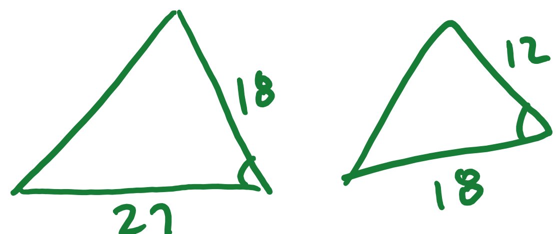
$\boxed{\text{yes, similar}}$

B. Are $\triangle PQS$ and $\triangle RQP$ similar?

One angle and adjoining sides. use SAS



$\angle Q \cong \angle Q$



$\frac{27}{18} = \frac{3}{2}$ $\frac{18}{12} = \frac{3}{2}$

yes, similar

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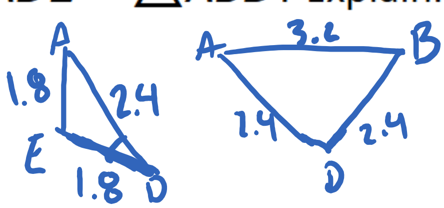
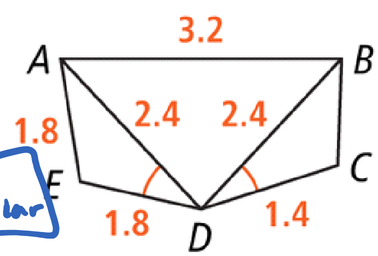
3. a. Is $\triangle ADE \sim \triangle ABD$? Explain.



← no angles use SSS

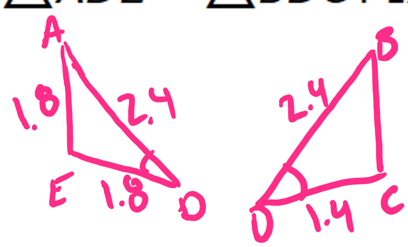


$\frac{1.8}{2.4} = \frac{3}{4}$

$\frac{2.4}{3.2} = \frac{3}{4}$

$\boxed{\text{similar}}$

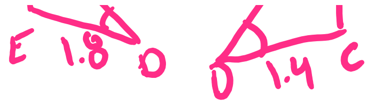
b. Is $\triangle ADE \sim \triangle BDC$? Explain.



one angle and 2 adjoining sides. SAS

$\frac{2.4}{2.4} = 1$ $\frac{1.8}{1.4} = \frac{9}{7}$

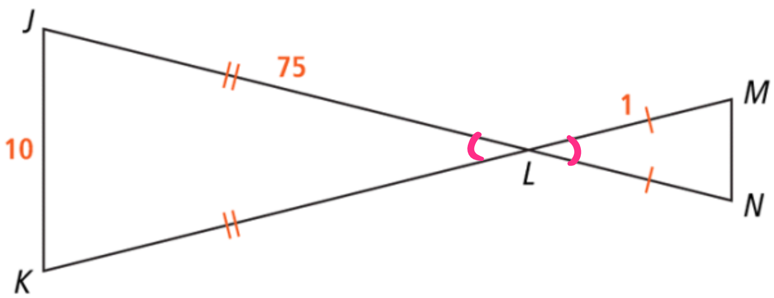
not similar



$\frac{2.4}{2.4} = 1$

$\boxed{\text{Not similar}}$

What is *MN*?

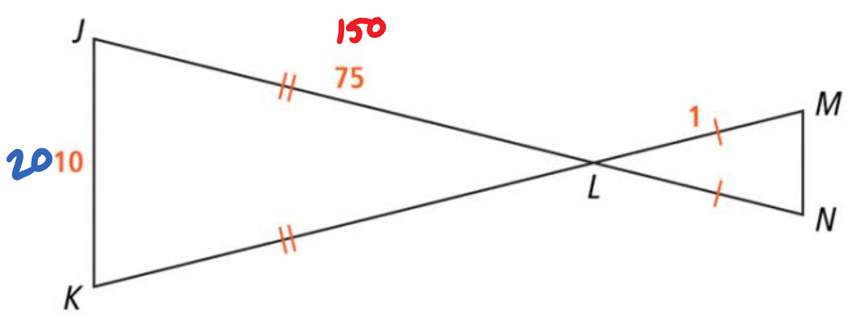


$\frac{1}{75} = \frac{x}{10}$

$x = \frac{10}{75} = \boxed{\frac{2}{15}}$

4. a. If the measure of $\overline{JL}$ were 150 instead of 75, how would the value of *MN* be different?

b. If the measure of $\overline{JK}$ were 20 instead of 10, how would the value of *MN* be different?

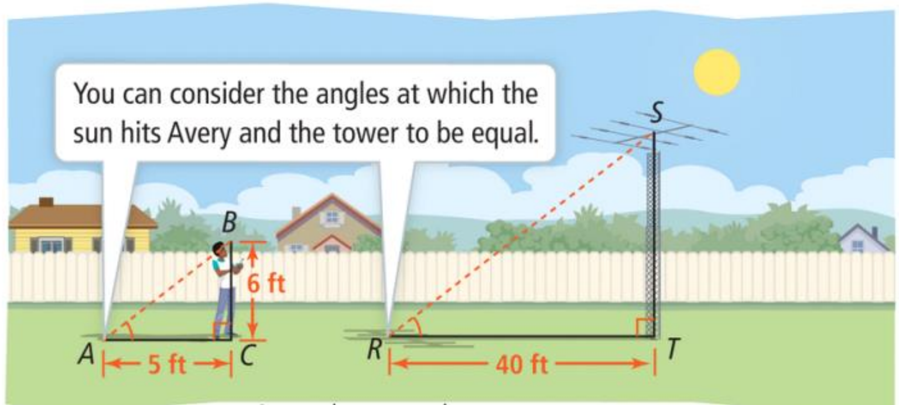


a) $\frac{150}{10} = \frac{1}{x}$ $\quad \frac{10}{150} = \frac{x}{1}$ $\quad \boxed{x = \frac{1}{15}}$

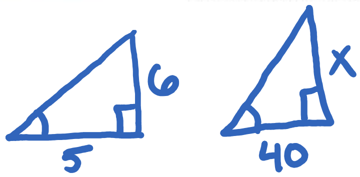
b) $\frac{x}{20} = \frac{1}{75}$ $\quad x = \frac{20}{75} = \boxed{\frac{4}{15}}$

Avery puts up a radio antenna tower in his yard. Ella tells him that their city has a law limiting towers to 50 ft in height. How can Avery use the lengths of his shadow and the shadow of the tower to show that his tower is within the limit without directly measuring it?

By using similar △'s.



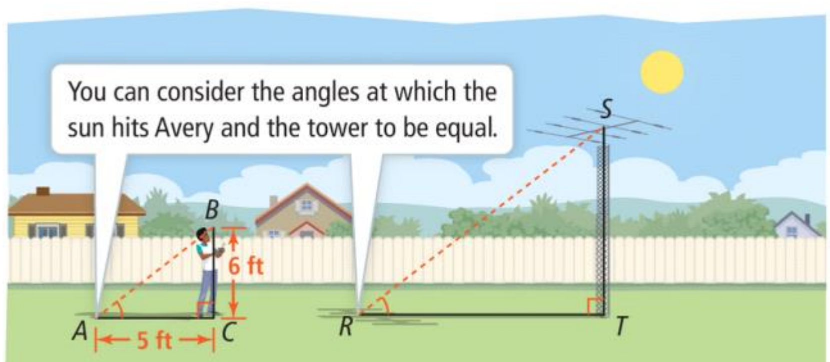
Art not drawn to scale.



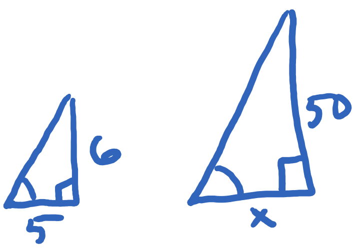
$$\frac{x}{40} = \frac{6}{5}$$

$$\boxed{x = 48}$$

5. If the tower were 50 ft tall, how long would the shadow of the tower be?



Art not drawn to scale.



$$\frac{x}{5} = \frac{50}{6}$$

$$x = \frac{250}{6} = \boxed{\frac{125}{3} \approx 41.\overline{6}}$$

https://tinyurl.com/rj68r7g

Homework

Pg. 322
10, 11, 16-20, 22, 26