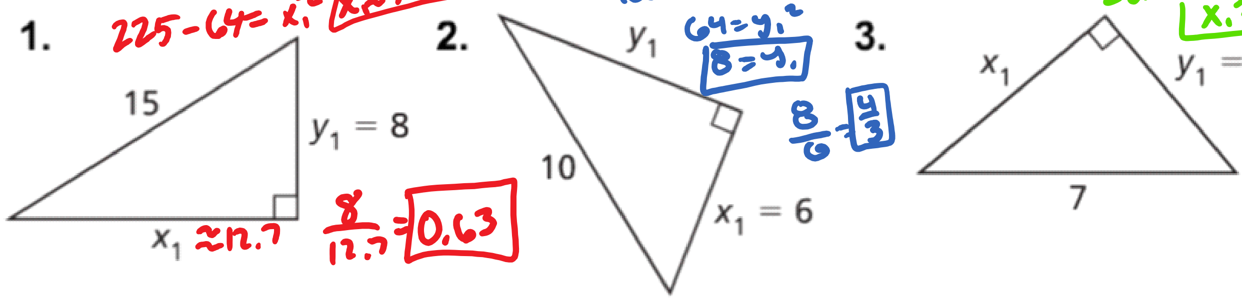
8.2 Trigonometric Ratios

Monday, September 23, 2019 7:46 AM

Warm Up

Find the measure of the missing leg in the right triangle, and then calculate the ratio $\frac{y_1}{x_1}$.



1. $15^2-8^2=x_1^2$
$225-64=x_1^2$
$\boxed{x\approx 12.7}$
$x_1 \approx 12.7$
$\frac{8}{12.7}=\boxed{0.63}$

2. $10^2-6^2=y_1^2$
$100-36=y_1^2$
$64=y_1^2$
$\boxed{8=y_1}$
$\frac{8}{6}=\boxed{\frac{4}{3}}$

3. $7^2-4.5^2=x_1^2$
$49-20.25=x_1^2$
$28.75=x_1^2$
$\boxed{x_1\approx 5.4}$
$\frac{4.5}{5.4}=\boxed{0.84}$

Essential Question

How do trigonometric ratios relate angle measures to side lengths of right triangles?

Needed Vocab:

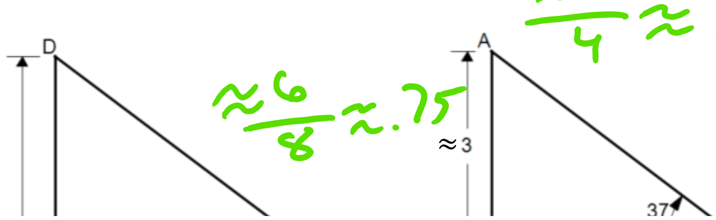
- **Cosine**
- **Inverse Ratios**
- **Sine**
- **Tangent**
- **Trigonometric Ratios**

Goal: "I can. . .

Use trigonometric ratios to find lengths and angle measures of right triangles."

Answer the following questions in order:

- Are the two triangles similar? yes.
- What is the approximate ratio between the side

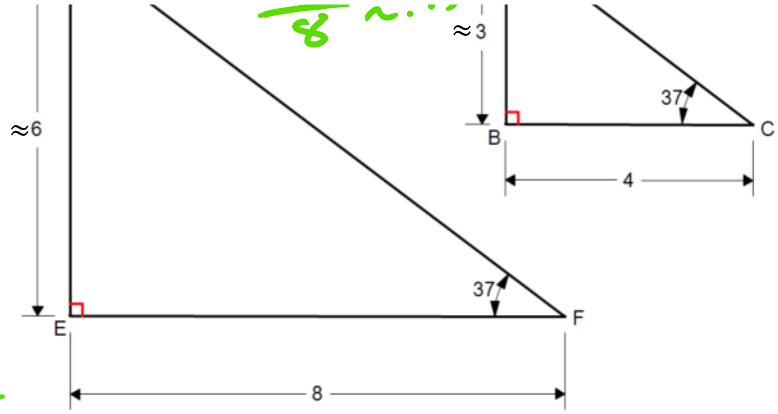


$\approx\frac{6}{8}\approx .75$

$\frac{\approx 3}{4}\approx .75$

- What is the approximate ratio between the side lengths of the triangle for each individual triangle?
- In your calculator, what is the number you get when you press tan(37)?

$\tan(37^\circ) \approx .75$



Trigonometric ratios are the consistent relationships between the side lengths of right triangles.

Sine compares the opposite side length over the hypotenuse of the triangle.

$$Sin(\theta) = \frac{Opp.}{Hyp.}$$

Cosine compares the adjacent side length over the hypotenuse of the triangle.

$$Cos(\theta) = \frac{Adj.}{Hyp.}$$

Tangent compares the opposite side length over the adjacent side length of the triangle.

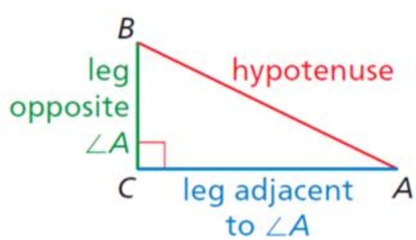
$$Tan(\theta) = \frac{Opp.}{Adj.}$$

Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written as $\sin A$ and $\cos A$) are defined as follows.

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

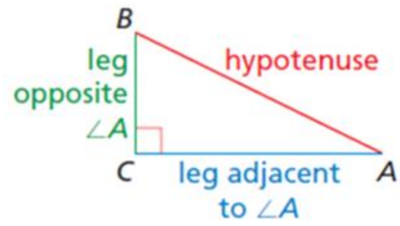


Tangent Ratio

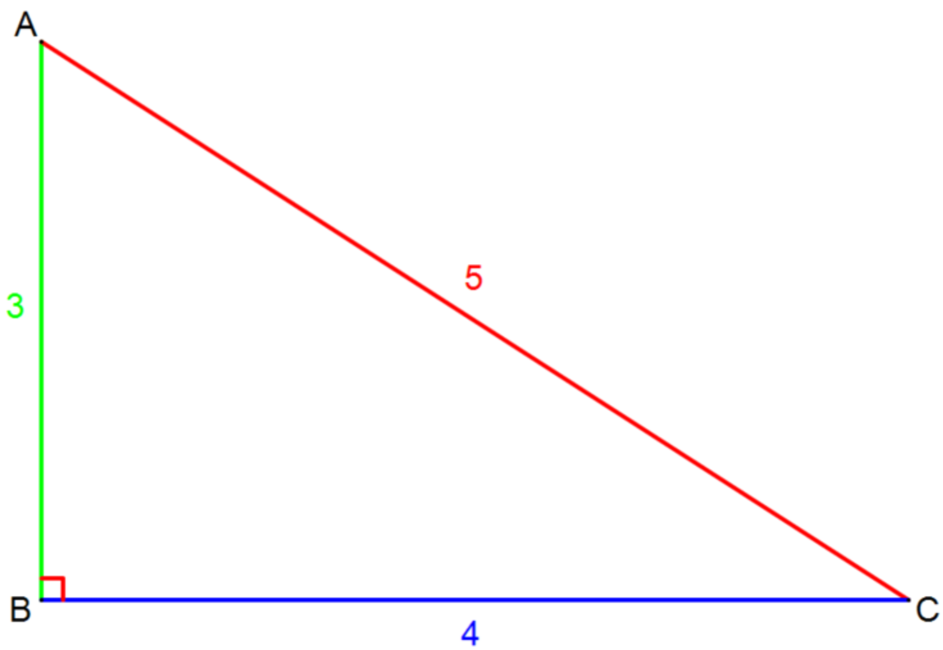
Let $\triangle ABC$ be a right triangle with acute $\angle A$.

The tangent of $\angle A$ (written as $\tan A$) is defined as follows.

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$



EXAMPLE 1 What are the Sine, Cosine, and Tangent ratios for angle A?

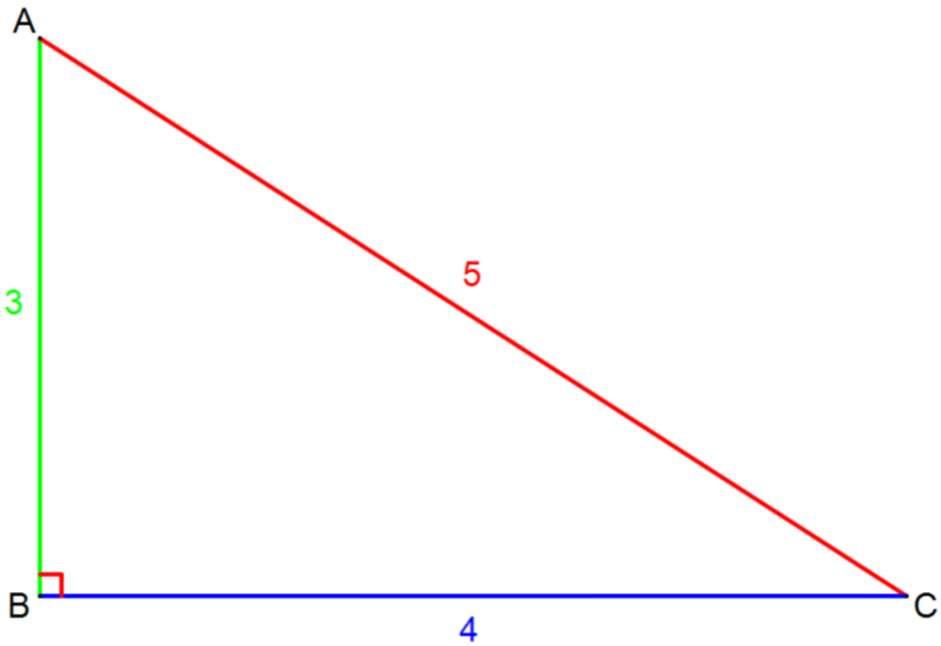


$$Sin(A) = \frac{4}{5}$$

$$Cos(A) = \frac{3}{5}$$

$$Tan(A) = \frac{4}{3}$$

What are the Sine, Cosine, and Tangent ratios for angle C?



$Sin(C) = \frac{3}{5}$

$Cos(C) = \frac{4}{5}$

$Tan(C) = \frac{3}{4}$

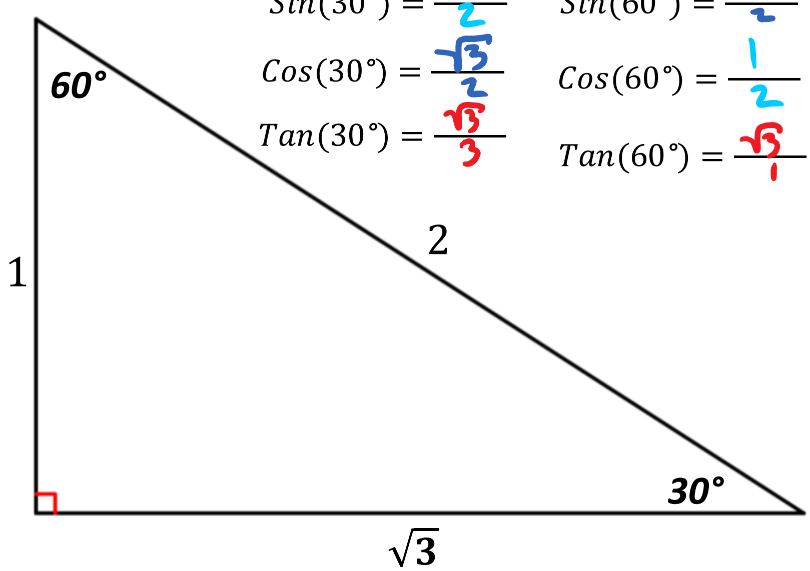
Example 2

What are the trig ratios of our special right triangles?

$Sin(30°) = \frac{1}{2}$ $\quad$ $Sin(60°) = \frac{\sqrt{3}}{2}$

$Cos(30°) = \frac{\sqrt{3}}{2}$ $\quad$ $Cos(60°) = \frac{1}{2}$

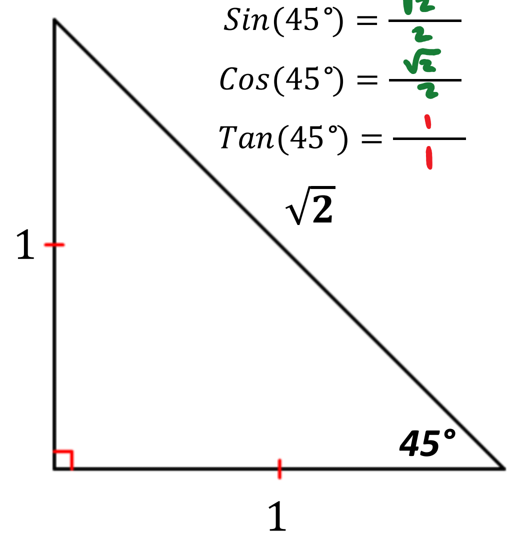
$Tan(30°) = \frac{\sqrt{3}}{3}$ $\quad$ $Tan(60°) = \frac{\sqrt{3}}{1}$



$Sin(45°) = \frac{\sqrt{2}}{2}$

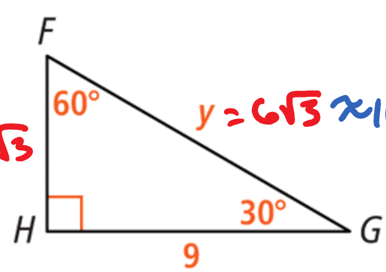
$Cos(45°) = \frac{\sqrt{2}}{2}$

$Tan(45°) = \frac{1}{1}$



a. In ΔFGH, what is the value of y?

30-60-90 Triangle calculation.



using trig.

$Cos(30°) = \frac{9}{y}$

$\downarrow$

$y = \frac{9}{cos(30°)}$

$\boxed{y \approx 10.4}$

b. How can you write an equivalent expression for cos 70° using sine? An equivalent expression for sin 34° using cosine?

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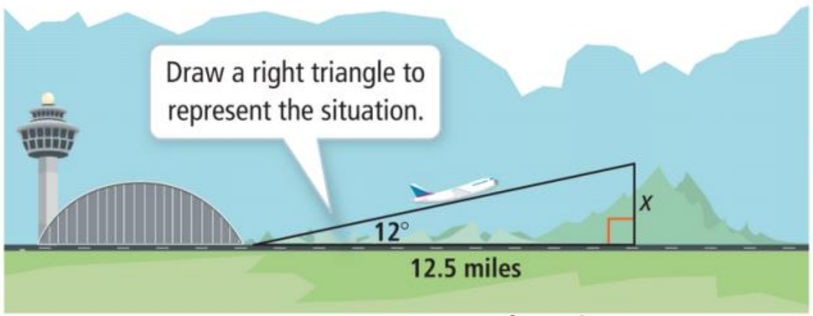
$\boxed{\cos(70^\circ) = \sin(20^\circ)}$

$\boxed{\sin(34^\circ) = \cos(56^\circ)}$

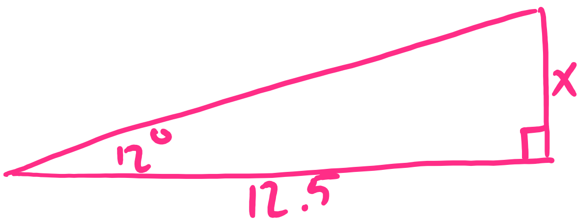
Example 3

A plane takes off and climbs at a 12° angle. Is that angle sufficient enough to fly over an 11,088-foot mountain that is 12.5 miles from the runway or does the plane need to increase its angle of ascent?



2.1 mi

1 mi = 5280 ft



$\tan(12^\circ) = \frac{x}{12.5}$

$x = 12.5 \cdot \tan(12^\circ)$

$x \approx 2.7$

yes, it will clear the mountain

If a plane climbs at 5° and flies 20 miles through the air as it climbs, what is the altitude of the plane, to the nearest foot?

$\sin(5^\circ) = \frac{x}{20}$

$x = 20 \cdot \sin(5^\circ)$

$x \approx 1.7$ miles

$\boxed{x \approx 9204 \text{ ft}}$

Inverse Ratios are used when you know the side lengths and need the angles.

$$Sin(\theta) = \frac{Opp.}{Hyp.} \qquad \theta = Sin^{-1}\left(\frac{Opp.}{Hyp.}\right)$$

$$Cos(\theta) = \frac{Adj.}{Hyp.} \qquad \theta = Cos^{-1}\left(\frac{Adj.}{Hyp.}\right)$$

$$Tan(\theta) = \frac{Opp.}{Adj.} \qquad \theta = Tan^{-1}\left(\frac{Opp.}{Adj.}\right)$$

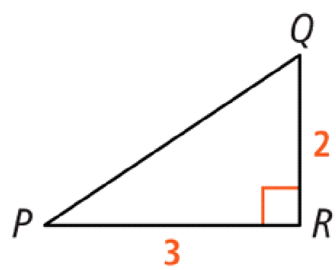
What is the m∠P?

$\tan(P) = \frac{2}{3}$

$P = \tan^{-1}\left(\frac{2}{3}\right)$

$\boxed{P \approx 33.7^\circ}$

What is the m∠Q?



$\tan(Q) = \frac{3}{2}$

$Q = \tan^{-1}\left(\frac{3}{2}\right)$

$\boxed{Q \approx 56.3^\circ}$

When solving for angle measures or side lengths in right triangles, what is given will dictate what trig ratio you should be using. Make sure to memorize those ratios.

https://tinyurl.com/scvjeu8

Homework

Pg. 359
21-39, 47, 48