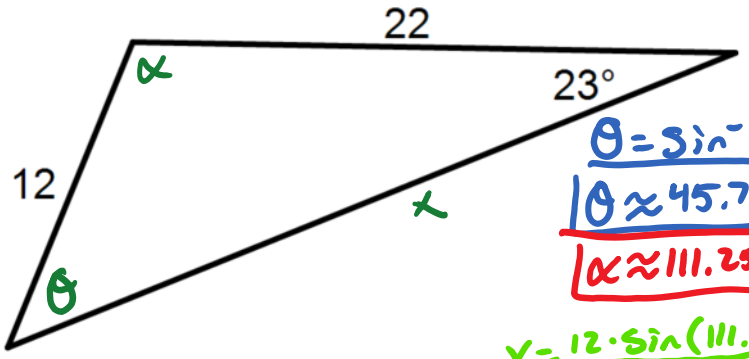
8.4 Law of Cosines

Monday, September 23, 2019 7:46 AM

Warm Up

Solve the following triangle.

$\frac{\sin(23)}{12} = \frac{\sin(\alpha)}{x} = \frac{\sin(\theta)}{22}$



$\theta = \sin^{-1}\left(\frac{22 \cdot \sin(23)}{12}\right)$

$\theta \approx 45.75°$

$\alpha \approx 111.25°$

$x = \frac{12 \cdot \sin(111.25)}{\sin(23)} \approx 28.6$

Essential Question

How can the Law of Cosines be used to determine side lengths and angle measures of acute and obtuse triangles?

Needed Vocab:

- **Law of Cosines**

Goal: "I can. . .

Use the Law of Cosines to solve problems."

Recall the table

$m\angle A$	a	$\frac{\sin(A)}{a}$	$m\angle B$	b	$\frac{\sin(B)}{b}$	$m\angle C$	c	$\frac{\sin(C)}{c}$
26.6°	$\sqrt{5}$	$1/5$	90°	5	$1/5$	63.4°	$2\sqrt{5}$	$1/5$

Fill in the following table with the correct information. What do you notice?

c	c^2	a	a^2	b	b^2	$m\angle C$	$a^2 + b^2 - 2ab \cdot \cos(C)$
$2\sqrt{5}$	20	$\sqrt{5}$	5	5	25	63.4°	≈ 20.0

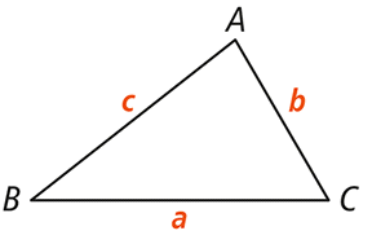
Law of Cosines

For any $\triangle ABC$, the **Law of Cosines** relates the cosine of each angle to the side lengths of the triangle.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

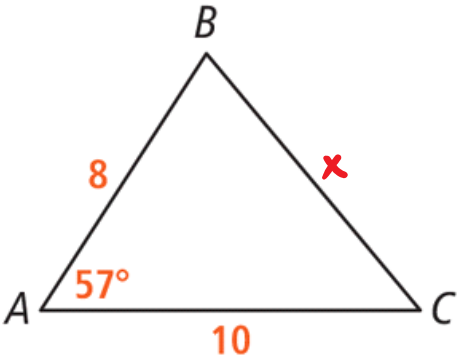
$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Example 1

What is *BC* to the nearest tenth?



Set up

$x^2 = 8^2 + 10^2 - 2(8)(10)\cos(57)$

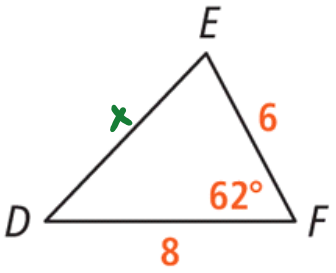
simplify down

$x^2 = 164 - 160\cos(57)$

plug into calc. (Simpler to go from right to left.)

$x \approx 8.8$

2. a. What is *DE*?

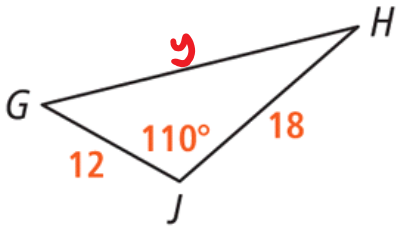


$x^2 = 8^2 + 6^2 - 2(8)(6)\cdot\cos(62)$

$x^2 = 100 - 96\cos(62)$

$x \approx 7.4$

b. What is *GH*?



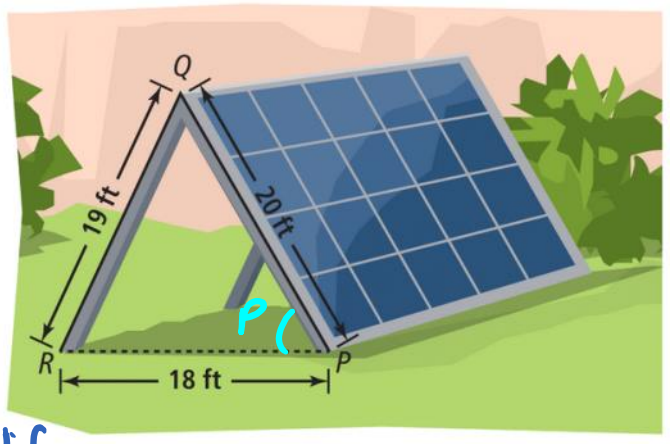
$y^2 = 12^2 + 18^2 - 2(12)(18)\cos(110)$

$y^2 = 468 - 432\cos(110)$

$y \approx 24.8$

Example 2

The optimal tilt for Keenan's solar panel is between 58° and 60° to the horizontal. Has Keenan placed his solar panel at an optimal angle?



Set up so side opp. the angle we want is solo.

$19^2 = 20^2 + 18^2 - 2(20)(18)\cos(P)$

$361 = 724 - 720\cos(P)$

Simplify

move the first over

$-724 \quad -724$

$-363 = -720\cos(P)$

Divide

$\frac{363}{720} = \cos(P)$

Find inverse

$P = \cos^{-1}\left(\frac{363}{720}\right)$

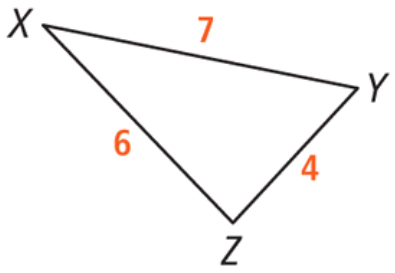
$P \approx 59.7$

Apply to situation.

Yes, it is set up correctly

3. a. What is $m\angle X$?

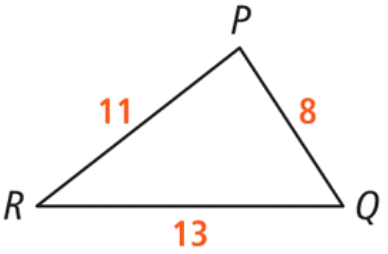
b. What is $m\angle P$?



$4^2 = 6^2 + 7^2 - 2(6)(7)\cos(x)$

$16 = 85 - 84\cos(x)$

$x \approx 34.8°$



$13^2 = 11^2 + 8^2 - 2(11)(8)\cos(P)$

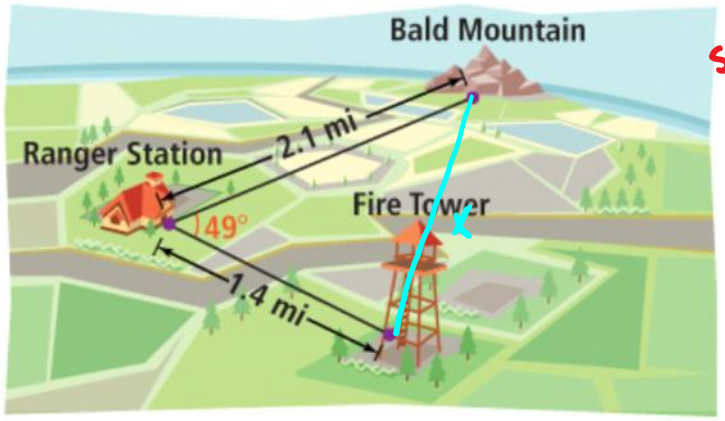
$169 = 185 - 176\cos(P)$

$P \approx 84.8°$

Example 3

The district ranger wants to build a new ranger station at the location of the fire tower because it would be closer to Bald Mountain than the old station is. Is the district ranger correct? Explain.

setup like before



$x^2 = 2.1^2 + 1.4^2 - 2(2.1)(1.4)\cos(49)$

Simp. $x^2 = 6.37 - 5.88\cos(49)$

Solve $x \approx 1.6$

Apply yes the ranger is correct

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4. Assume a path is drawn from the fire tower to Bald Mountain. What is the angle the new path forms with the old path from Bald Mountain to the ranger station?

Using Law of Cosines

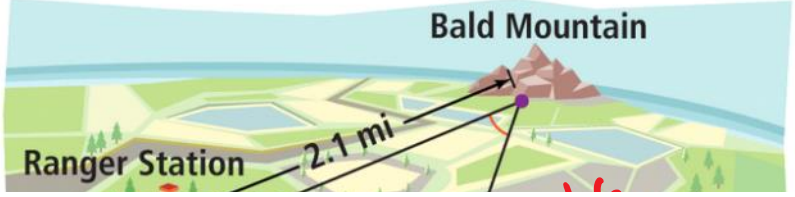
$1.4^2 = 1.6^2 + 2.1^2 - 2(1.6)(2.1)\cos(x)$

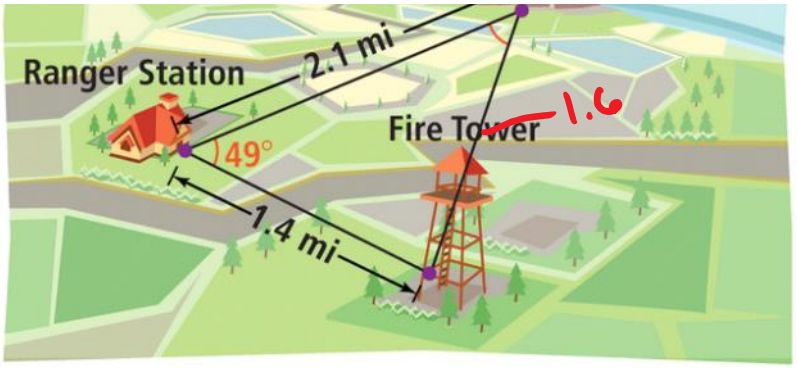
$1.96 = 6.97 - 6.72\cos(x)$

$x \approx 41.8°$

Since we have an opp. pair (49° and 1.6) we can use Law of Sines.

$\frac{\sin(49)}{1.6} = \frac{\sin(x)}{2.1}$





Use Law of Sines.

$$\frac{\sin(49)}{1.6} = \frac{\sin(x)}{1.4}$$

$$x = \sin^{-1}\left(\frac{1.4 \cdot \sin(49)}{1.6}\right)$$

$$x \approx 41.3^\circ$$

Since the 1.6 is an approx. it makes them differ.

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https://tinyurl.com/wj53gzy

Homework

Pg. 371
14, 17-28, 30, 33