## UNDERSTAND

19. Make Sense and Persevere Describe two ways to express the edge length of a cube with a volume shown.

20. Construct Arguments Explain why $5^{\frac{4}{3}}$ must be equal to $\sqrt[3]{5^{4}}$ if the Power of a Power Property holds for rational exponents.
21. Error Analysis Describe and correct the error a student made when starting to solve the equation $8^{x+3}=2^{2 x-5}$.

$$
\begin{aligned}
8^{x+3} & =2^{2 x-5} \\
\left(2^{3}\right)^{x+3} & =2^{2 x-5} \\
2^{3 x+3} & =2^{2 x-5} \\
& \vdots
\end{aligned}
$$

22. Construct Arguments The Power of a Quotient rule is $\left(\frac{a}{b}\right)^{m}=\frac{a^{m}}{b^{m}}, b \neq 0$. Will this rule work with rational exponents if $\frac{a}{b}$ is a positive number? Give an example to support your argument.
23. Higher Order Thinking The Zero Exponent Property is $a^{0}=1, a \neq 0$.
a. How could you use properties of exponents to explain why $a^{0}=1$ ?
b. How could the Zero Exponent Property be applied when solving equations with rational exponents?
24. Use Structure Consider the expression $\sqrt{\sqrt{625}}$.
a. Write the radical using rational exponents.
b. Describe two different ways to evaluate the expression.
c. Simplify the expression from part (b).

## PRACTICE

Write each radical using rational exponents.
SEE EXAMPLE 1
25. $\sqrt{3}$
26. $\sqrt[3]{7}$
27. $\sqrt[5]{3^{2}}$
28. $\sqrt[4]{2^{-5}}$
29. $\sqrt[3]{a^{2}}$
30. $\sqrt{b^{a}}$

Solve each equation. See examples 2-5
31. $\left(5^{\frac{x}{3}}\right)\left(5^{\frac{x}{4}}\right)=5^{5}$
32. $\left(2^{\frac{x}{2}}\right)\left(4^{\frac{x}{2}}\right)=2^{6}$
33. $\left(3^{\frac{x}{2}+1}\right)=\left(3^{-\frac{5 x}{2}}\right)$
34. $625^{2 x-3}=25^{3 x-2}$
35. $\left(\frac{1}{243}\right)^{-\frac{x}{3}}=\left(\frac{1}{9}\right)^{-\frac{x}{2}+1}$
36. $8^{\frac{-x}{3}}=4$
37. $49^{\frac{x}{4}-1}=343^{\frac{x}{3}}$
38. $3=\left(5^{\frac{1}{2}}\right)\left(x^{\frac{1}{2}}\right)$
39. $2=\left(4^{\frac{1}{3}}\right)\left(2^{\frac{x}{3}}\right)$
40. $\frac{27^{\frac{1}{4}}}{3^{\frac{x}{4}}}=1$
41. $5^{-\frac{2}{3}}=\frac{125^{\frac{x}{3}}}{25^{\frac{4}{3}}}$
42. $\frac{6^{\frac{1}{4}}}{36^{-\frac{x}{2}}}=1$

For each partial solution, identify the property of exponents that is used. See examples 2-4
43.

$$
\begin{aligned}
36^{\frac{x}{3}+3} & =216^{\frac{x}{5}} \\
\left(6^{2}\right)^{\frac{x}{3}+3} & =\left(6^{3}\right)^{\frac{x}{5}} \\
6^{\frac{2 x}{3}+6} & =6^{\frac{3 x}{5}}
\end{aligned}
$$

44. 

$$
\begin{aligned}
\frac{3^{\frac{3 x}{4}}}{3^{\frac{1}{4}}} & =3^{-\frac{3}{4}} \\
3^{\frac{3 x}{4}-\frac{1}{4}} & =3^{-\frac{3}{4}}
\end{aligned}
$$

## APPLY

45. Use Appropriate Tools The formula for the volume $V$ of a sphere is $\frac{4}{3} \pi r^{3}$. What is the radius of the basketball shown?

46. Use Structure A singing contest eliminates contestants after each round. To find the number of contestants in the next round, raise the number of contestants in the current round to the power of $\frac{6-n}{7-n^{\prime}}$ where $n$ is the number of the current round.
47. Make Sense and Persevere Photos A, B, and $C$ are all square photos. The area of Photo $C$ is the same as a rectangular photo whose length is the side length of Photo $A$ and whose width is the side length of Photo B. Use the properties of rational exponents to write and solve an equation to find the side length of Photo $A$ to two decimal places.


Photo A
Area $=x \mathrm{~cm}^{2}$


Photo B
Area $=72 \mathrm{~cm}^{2}$


Photo C Area $=110 \mathrm{~cm}^{2}$

## ASSESSMENT PRACTICE

48. Match each expression with its equivalent expression.
I. $\sqrt[4]{2^{5}}$
A. $2^{\frac{1}{5}}$
II. $\sqrt{5}$
B. $2^{\frac{5}{4}}$
III. $\sqrt[5]{2^{4}}$
C. $2^{\frac{4}{5}}$
IV. $\sqrt[5]{2}$
D. $5^{\frac{1}{2}}$
49. SAT/ACT What is the value of $x$ in $27^{\frac{x}{2}}=3^{x-1}$ ?
(A) -3
(B) -2
(C) $\frac{1}{3}$
(D) 2
(E) 3
50. Performance Task It is possible to write any positive integer as the sum of powers of 2 with whole number exponents. For example, you can write 75 in the following manner.

$$
2^{0}+2^{1}+2^{3}+2^{6}=75
$$

Part A Use the equation above to write 75 as the sum of powers of 8 , using rational exponents. What are possible values for $a, b$, $c$ and $d$ ?

$$
8^{a}+8^{b}+8^{c}+8^{d}=75
$$

Part B How can you modify the equation you wrote in part A to express 75 as sum of powers of 16 ?

$$
16^{a}+16^{b}+16^{c}+16^{d}=75
$$

Part C Given that $a, b, c$, and $d$ are rational numbers, for what types of integer values of $x$ is the following equation true? Explain your answer.

$$
x^{a}+x^{b}+x^{c}+x^{d}=75
$$

