



Selected Answers

Topic 1

Lesson 1-1

2. Ella found the sum of the coordinates instead of the difference of the coordinates. **4.** 42° **6.** 9 **8.** 47°

10. By the Segment Addition Postulate, $AE = AC + CE$, $AC = AB + BC$, and $CE = CD + DE$, so using substitution, $AE = AB + BC + CD + DE$. **12.** Answers may vary. Sample: Benito used equal signs where he should have used the congruency symbol. The segments and angles are congruent, but their lengths and measures are equal. **14.** $\frac{7}{18}$ **16.** 3

18. 3 **20.** $1\frac{1}{2}$ **22.** 3 **24.** 14 **26.** 35°

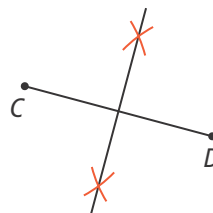
28. 13° **30.** 11 **32.** 28° **34.** 3

36. Springfield; Since the distance to Gilmore is 26 miles, half way is 13 miles. The closest town to 13 miles away is Springfield. **38.** 3; Sample: Since $234 - 52 - 68 = 114$, the total length of the four segments with tick marks is 114 feet. There are two segments with one tick mark and two sides with two tick marks, and the length of Dayton Avenue is the sum of the length of a segment with one tick mark and the length of a segment with two tick marks. The total length along Dayton Avenue is half of 114 feet, or 57 feet. $57 \div 20 \approx 60 \div 20 = 3$. **40.** E

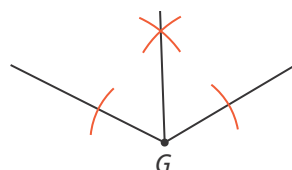
Lesson 1-2

2. Chris did not use the compass to measure the distance from where the arc intersects one side of $\angle T$ to where it intersects the other side. **4.** Copy \overline{AC} using D as one endpoint and using the side of $\angle D$ that is not \overline{DE} . Label the other endpoint F . Connect E and F .

6.



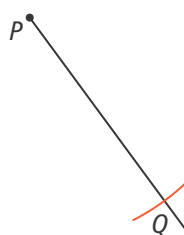
8.



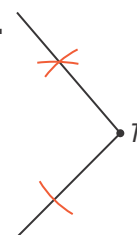
10. Open the compass to the length of one of the segments. Without changing the setting, place the compass point at one of the endpoints of the other segment and check if the pencil point of the compass can draw an arc through the other endpoint. **12.** $\frac{n}{6}$

14. Adam made an arc from one side of the angle to the other side. He should have made an arc across the angle from R and used the intersection of this arc with the sides as the center for the arcs in the center of the angle.

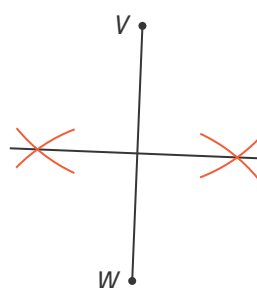
16.



18.



20.

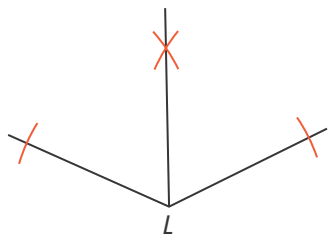




Selected Answers

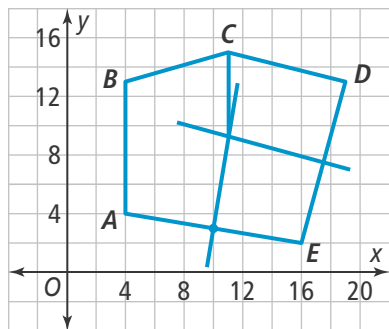
Topic 1

22.



24. Construct the perpendicular bisector of \overline{AB} . Let E be the midpoint of \overline{AB} and F be the point where the perpendicular bisector and \overline{DC} intersect. Construct the bisectors of $\angle AEF$ and $\angle BEF$. Let G and H be the points where the angle bisectors intersect the sides of the square. Construct the bisectors of $\angle GEF$ and $\angle HEF$.

26. about (10, 9)



28. C

Lesson 1-3

2. Corey added the x - and y -coordinates of each point rather than adding the x -coordinates of both points and the y -coordinates of both points. 4. No; if (a, b) and (c, d) are midpoints, then $a = \frac{x_1 + x_2}{2} = c$ and $b = \frac{y_1 + y_2}{2} = d$, so $a = c$ and $b = d$. 6. (3, -2) 8. 187 feet 10. The student did not add the x - and y -coordinates when calculating the midpoint. The correct answer is

$$M\left(\frac{-4 + (-1)}{2}, \frac{5 + (-4)}{2}\right), \text{ or } M\left(-\frac{5}{2}, \frac{1}{2}\right).$$

12. a. (4, 10) b. Sample: Find the point 4 times further from the origin than the given point, $Q(8, 20)$.

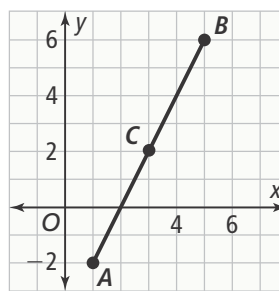
$$14. PM = \sqrt{(a - c)^2 + (b - d)^2}$$

$$PQ = 2PM = 2\sqrt{(a - c)^2 + (b - d)^2}$$

16. (8, 2) 18. (-0.3, 3.25) 20. 29.2 yards

22. Arthur is closest, about 29.2 yards away. Cameron is farther, about 36.0 yards, and Jamie is farthest, about 41.2 yards. 24. approximately 29.15

26.



28. **Part A** On Broadway, the parade travels about 6.71 units. On Central Avenue, the parade travels about 5.66 units. The total distance is 12.37, or 3.10 miles. **Part B** Broadway and Cedar St. **Part C** (8, 1) and (4, 2)

Lesson 1-4

2. Every triangle has 3 altitudes and Esteban found only 1 altitude for each triangle. If he had found all 3 altitudes for the obtuse triangle, he would have discovered that the other 2 altitudes lie outside of the triangle. 4. 32, 39, 46 6. Answers may vary. Sample: A counterexample is $n = 3$ because $32 + 1 = 10$, and 10 is even. 8. Danielle only gave one true example, which is not enough to support a conjecture. In fact, the statement is false. As a counterexample, vertical angles share a vertex and are not adjacent.



Selected Answers

Topic 1

10. 53, 41, 29 **12.** The number of triangles is $n - 2$. **14.** Answers should be between 264 and 270. **16.** Answers may vary. Sample: Both 8 and 0 are rational numbers, but $8 \div 0$ is not a rational number. This is a counterexample, so the statement is false. **18.** Deshawn could start by finding all three-digit numbers where the first and third digits are equal to each and the sum of the first and third digits is equal to the second digit. Then he could determine if each is divisible by 11.

$$121(1 + 1 = 2): 121 \div 11 = 11;$$

$$242(2 + 2 = 4): 242 \div 11 = 22;$$

$$363(3 + 3 = 6): 363 \div 11 = 33;$$

$$484(4 + 4 = 8): 484 \div 11 = 44$$

Since $5 + 5 = 10$, there are no more three-digit numbers that fit this description. Since the statement has been shown to be true for all possible cases, the statement is true.

20.

	Yes	No
12	<input type="checkbox"/>	<input checked="" type="checkbox"/>
19	<input type="checkbox"/>	<input checked="" type="checkbox"/>
22	<input checked="" type="checkbox"/>	<input type="checkbox"/>
28	<input type="checkbox"/>	<input checked="" type="checkbox"/>
30	<input checked="" type="checkbox"/>	<input type="checkbox"/>

22. B

Lesson 1-5

2. She negated the hypothesis but not the conclusion. **4.** Answers may vary. Sample: To write a converse, switch the hypothesis and conclusion. To write a

contrapositive, negate the hypothesis and conclusion and then switch them.

6. A number is prime if and only if it has only 1 and itself as factors. **8.** True; Answers may vary. Sample: A rectangle with sides of length 2 m and 6 m.

10. If the city can build new roads, then it raised the sales tax to 10%. If the city raised the sales tax to 10%, then it can build new roads. **12.** Answers may vary. Sample: A number is even if and only if it is divisible by 2. If a number is even, then it is divisible by 2. If a number is divisible by 2, then it is even.

14. Answers may vary. Sample:

If \overrightarrow{BX} is the bisector of $\angle ABC$, then $\angle ABX \cong \angle XBC$.

16. The hypothesis and conclusion have the same truth value. **18.** Conditional: If $\triangle XYZ$ is a right triangle, then $a^2 + b^2 = c^2$.

Biconditional: Triangle XYZ is a right triangle if and only if $a^2 + b^2 = c^2$.

20. If a number is divisible by 6, then it is divisible by 3. **22.** True; if the hypothesis is true and a pair of lines is parallel, then they will never intersect, so the conclusion is also true.

24. Negation of hypothesis: The sum of the interior angle measures of a polygon is not 180° ; Negation of conclusion: The polygon is not a triangle.

26. If an angle is obtuse, then it measures 100° . This is false because a counterexample is a 99° angle.

28. If an angle does not measure 100° , then it is not obtuse. This is false because a counterexample is a 99° angle. **30.** If a month has exactly 28 days, then it is February. If a month is February, then it has exactly 28 days.

32. If the area of a square is s^2 , then



Selected Answers

Topic 1

the perimeter of the square is $4s$.

If the perimeter of a square is $4s$, then the area of the square is s^2 .

34. a. Sample conditional: If the year was 1881, then three different men were President of the United States. Sample biconditional: The year was 1881 if and only if three different men were President of the United States.

b. The conditional is true, but the biconditional is false.

36.

Statement	Truth Value
Conditional	F
Converse	T
Inverse	T
Contrapositive	F

38. Part A Answers may vary.

Sample: If a triangle is a right triangle, then two angles are complementary. **Part B** Students should create several right triangles and find that, in every case, the acute angles are complementary.

Part C Converse: If two angles in a triangle are complementary, then the triangle is a right triangle. Inverse: If a triangle is not a right triangle, then two angles are not complementary.

Contrapositive: If two angles in a triangle are not complementary, then the triangle is not a right triangle.

Biconditional: A triangle is a right triangle if and only if two angles are complementary. The biconditional is true because the angles in a triangle sum to 180° , so if any two angles are complementary, then the third angle must be $180^\circ - 90^\circ = 90^\circ$.

Lesson 1-6

2. The statement "My team won the championship" is the conclusion of the conditional, not the hypothesis. Dakota cannot apply the Law of Detachment.

4. Answers may vary. Sample: Using symbols makes it easier to see if the structure of the Law of Detachment can be applied ($p \rightarrow q$ and p are true) or if the structure of the Law of Syllogism can be applied ($p \rightarrow q$ and $q \rightarrow r$).

6. $AC = 8x - 3$ **8.** If two numbers are odd, the sum is divisible by 2.

10.

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

12. The hypothesis must be false. Get the contrapositive of the conditional by negating the hypothesis and conclusion. The contrapositive is equivalent to the conditional so it is true. The hypothesis of the contrapositive is the negation of the original conclusion so it is true. By the Law of Detachment, the conclusion of the contrapositive is true. This is the negation of the original hypothesis, so the original hypothesis must be false.

14. true **16.** The coordinates of the midpoint are $(-\frac{1}{2}, \frac{1}{2})$. **18.** If ray \overrightarrow{BD} bisects angle $\angle ABC$, then $m\angle ABD = m\angle DBC$. **20.** If it is Tuesday night,



Selected Answers

Topic 1

Zachary goes to bed early. **22.** If the endpoints of a segment are $P(x_1, y_1)$ and $Q(x_2, y_2)$, then the distance from P to the midpoint is $\frac{1}{2}PQ$. The length of the segment is $PQ = 10$. The distance from P to the midpoint of \overline{PQ} is 5.

24. a. If you win a magnet or stickers, you win a prize in category A. If you win a keychain or flashlight, you win a prize in category B. If you win earbuds or an MP3 speaker, you win a prize in category B. If you win a prize in category A, you spend 0 to 100 tickets. If you win a prize in category B, you spend 101 to 200 tickets. If you win a prize in category C, you spend 201 to 300 tickets. **b.** If Ines wins an MP3 speaker, she spends 201 to 300 tickets. Ines wins a prize in category C. Ines spends 201 to 300 tickets. **26.** false, true, true, false **28. Part A** She has 3 blue chips, 1 red chip, and 1 green chip. **Part B** 1, 2, 2, 5

Lesson 1-7

2. Jayden should have used the Congruent Complements Theorem.

4. Sample: Since $\angle 2$ and $\angle 3$ are complementary by the Vertical Angles Theorem, $\angle 1$ and the angle labeled 30° are complementary, which means the sum of their measures is 90° . Therefore, $m\angle 1 = 90^\circ - 30^\circ = 60^\circ$.

6. $\angle ADC \cong \angle EDG$; the Vertical Angles Theorem says vertical angles are congruent, not that their measures are equal. **8.** $x^\circ + 4^\circ = 90^\circ$ **10.** The student mistakenly used the definition of complementary angles instead of the definition of supplementary angles.

12. Statements	Reasons
1) $\angle ABC$ and $\angle CBD$ are a linear pair.	1) Given
2) $m\angle ABC + m\angle CBD = 180^\circ$	2) Linear Pairs Thm.
3) $\angle ABC$ and $\angle CBD$ are supplementary.	3) Def. suppl. angles

14. $x = 42$; 84° ; 84° **16.** $x = 55$; 118° ; 118° **18.** By the Angle Addition Postulate, $m\angle DHF = 25^\circ + 41^\circ$, which simplified is $m\angle DHF = 66^\circ$. By the definition of supplementary angles, $\angle DHF$ is supplementary to $\angle ABC$. Since $\angle GHF$ is also supplementary to $\angle ABC$, by the Congruent Supplementary Theorem, $\angle DHF \cong \angle GHF$.

20. Statements	Reasons
1) $m\angle 2 = 30^\circ$	1) Given
2) $m\angle 1 = 2m\angle 2$	2) Given
3) $m\angle 1 = 2(30^\circ)$	3) Subst.
4) $m\angle 1 = 60^\circ$	4) Simplify.
5) $m\angle 3 = 30^\circ$	5) Vert. Angles Thm.
6) $m\angle 4 = 60^\circ$	6) Vert. Angles Thm.
7) $m\angle 3 + m\angle 4 = 30^\circ + m\angle 4$	7) Add Prop.
8) $m\angle 3 + m\angle 4 = 30^\circ + 60^\circ$	8) Subst.
9) $m\angle 3 + m\angle 4 = 90^\circ$	9) Simplify.

22. $m\angle 2 = 50^\circ$, $m\angle 4 = 85^\circ$, $m\angle 5 = 50^\circ$, $m\angle 6 = 45^\circ$ **24.** always true, never true, sometimes true, sometimes true, always true, sometimes true **26. Part A** From the top angle clockwise, the measures are 96° , 51° , 33° , 96° , 51° , and 33° .



Selected Answers

Topic 1

Part B Answers may vary. Sample:

Statements	Reasons
1) $m\angle AGB = 33^\circ$ and $m\angle AGF = 51^\circ$	1) Given
2) $m\angle CGC = 51^\circ$	2) Vert. Angles Thm.
3) $m\angle DGE = 33^\circ$	3) Vert. Angles Thm.

Lesson 1-8

2. One method is to prove by contradiction, and another is to prove the contrapositive. Given the conditional $p \rightarrow q$, both methods start by assuming the negation of q .

4. The following are truth tables for a conditional and a contrapositive:

p	q	$p \rightarrow q$	$\sim p \rightarrow \sim q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Since the conditional and the contrapositive have the same truth table, proving the contrapositive is equivalent to proving the conditional.

6. It did not rain on Thursday.

8. Assume $m\angle JKM \neq m\angle JKL - m\angle MKL$.

10. Answers may vary. Sample: Since Nadeem spent \$8.49 on the sandwich, he spent between \$1.52 and \$2.50 on his drink. Apple juice, tropical smoothie, and milk do not fall in that range. The only remaining choice is orange juice. Nadeem drank orange juice with his sandwich. **12.** Answers may vary. Sample: Assume $x > 3$. Then $3x > 9$, so $3x$ cannot be less than 10. This contradicts the original statement, so $x \leq 3$. An indirect proof by

contradiction is easier because a proof by contrapositive involves more steps.

14. She should have assumed x is not even, or x is odd, and then written it in the form $2k + 1$. **16.** Derek did not get an A on the history test. **18.** \overline{DE} is not the angle bisector of $\angle ADC$. **20.** II and III

22. Contrapositive: If $EF \neq 14$, then $AB + CD \neq 14$. Assume $EF \neq 14$. Then $EF > 14$ or $EF < 14$. First consider $EF > 14$. $AB + CD = 6 + 8 = 14$, so $EF > AB + CD$, which proves this case of the contrapositive. Next consider $EF < 14$. Again $AB + CD = 6 + 8 = 14$, so $EF < AB + CD$, which proves this case of the contrapositive. Therefore, the original conditional must be true.

24. Contrapositive: If more than two of the friends have more than one slice of pie each, then the number of friends who share the pie is not four. Assume more than two of the friends have more than one slice of pie each. Then at least three of them have two or more slices of pie, which is at least six slices of pie. Then the number who share the pie cannot be four, which proves the contrapositive. Therefore the conditional is true. **26.** Yes; If $\angle P$ and $\angle Q$ are obtuse angles, then $m\angle P > 90^\circ$ and $m\angle Q > 90^\circ$. If $\angle P$ and $\angle Q$ are supplementary, then $m\angle P + m\angle Q = 180^\circ$. But, $m\angle P + m\angle Q > 90^\circ + 90^\circ = 180^\circ$.

28. Part A Assume a customer chooses a meal that does not have a cheese sandwich. Then the customer has either a ham sandwich or a chicken salad sandwich. Neither of these meals includes a banana, so the customer does not have a banana. This proves the contrapositive, so the original



Selected Answers

Topic 1

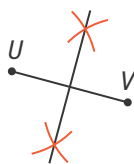
statement is true: If a customer chooses a meal that has a banana, then the customer also has a cheese sandwich.

Part B If a customer chooses a meal that has yogurt, then the customer does not have an apple. Proof: Assume the customer has an apple. Then the customer must have chosen the meal that has the cheese sandwich, carrot sticks, and apple. Therefore the customer does not have yogurt. This proves the contrapositive, so the original statement is true: If a customer chooses a meal that has yogurt, then the customer does not have an apple.

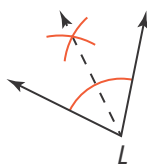
Topic Review

2. postulate 4. biconditional
6. theorem 8. deductive reasoning
10. 63 12. 32° 14. -9

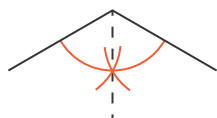
16.



18.



20.



22. $(3, 2)$; $2\sqrt{5}$ 24. $(-1, \frac{1}{2})$; 5

26. $(11, 7\frac{1}{2})$ 28. 15, 18 30. 37, 41

32. 2 and 14; 4 and 16; 6 and 18; 8 and 20 34. \$17.50 36. Conditional: If it is Saturday, then Kona jogs 5 miles. Converse: If Kona jogs 5 miles, then it is Saturday. Inverse: If it is not Saturday, then Kona does not jog 5 miles. Contrapositive: If Kona does not jog 5 miles, then it is not Saturday.
38. True; simplifying the inequality gives, and any number less than is also less than 8. 40. Pudding is available if and only if it is a Tuesday. 42. If it is a sunny day, the lines for each ride are long. 44. $x = 28$; 78° ; 78°

46.

Statements	Reasons
1. $m\angle TUV = 90^\circ$	1. Given
2. $m\angle TUW + m\angle WUV = m\angle TUV$	2. Angle Addition Postulate
3. $y^\circ + 42^\circ = 90^\circ$	3. Substitution Property
4. $y^\circ = 48^\circ$	4. Subtraction Property
5. $4x = y$	5. Vertical Angle Theorem
6. $4x = 48$	6. Substitution Property
7. $x = 12$	7. Simplify.



Selected Answers

Topic 2

Lesson 2-1

2. alternate interior angles, alternate exterior angles, and corresponding (and vertical) angles **4.** The angles can be congruent or supplementary.

6. supplementary, Same-Side Interior Angles Postulate **8.** 109°

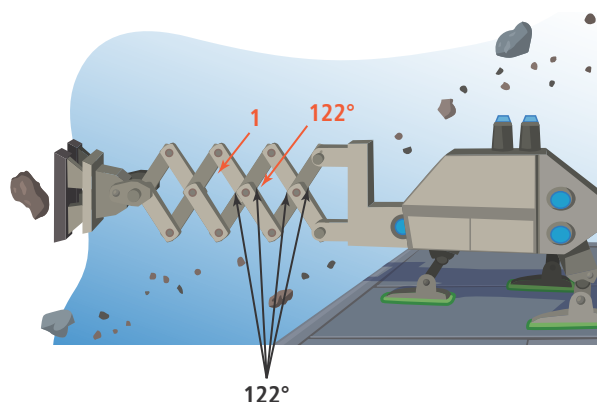
10. Answers may vary. Sample:

Statements	Reasons
$m \parallel n$	Given
$\angle 1 \cong \angle 3$	Corresponding Angles Theorem
$\angle 3 \cong \angle 2$	Vertical Angles Theorem
$\angle 1 \cong \angle 2$	Transitive Property of Congruence

12. While there is no Same-Side Exterior Angles Theorem given, same-side exterior angles are supplementary, not congruent. $m\angle 1 = 180^\circ - 72^\circ = 108^\circ$

14. $w = 15$, $v = 6$ **16.** Answers may vary. Sample: $\angle 7$ and $\angle 3$ **18.** Angles 2, 3, 6, and 7 are supplementary. Angles 4, 5, and 8 are congruent. **20.** 57°

22. $m\angle 1 = 58^\circ$. All of the angles along the center of the parallelogram linkage are congruent either because they are vertical angles or because they are opposite angles of a parallelogram. Since all of the arms of the linkages are parallel, $\angle 1$ is supplementary to these angles. Therefore, $m\angle 1 = 180^\circ - 122^\circ = 58^\circ$.



24. $m\angle 1 = 61^\circ$ because it is a corresponding angle to the desired angle. Angles 1 and 2 are same-side interior angles, so they are supplementary. Therefore, $m\angle 2 = 180^\circ - 61^\circ = 119^\circ$.

26. 125° , 55° **28.** B

Lesson 2-2

2. She should have written $m\angle 1 + m\angle 2 = 180^\circ$. **4.** The Converse of the Corresponding Angles Theorem applies to corresponding angles with any measure, but Theorem 2-9 applies to corresponding angles that are both right angles. **6.** 12; Converse of the Alternate Exterior Angles Theorem **8.** Assume ℓ is not parallel to m .

Construct line $n \parallel l$.

$m\angle 3 > 0$, by the assumption.

$m\angle 1 = m\angle 2 + m\angle 3$ by Converse Alternate Interior Angles Theorem,
 $m\angle 1 = m\angle 2$ is given. Contradiction.

The assumption is false. $l \parallel m$.

10. By the Angle Addition Postulate, $m\angle 1 + m\angle 2 = 43^\circ + 40^\circ = 83^\circ$ and $m\angle 3 + m\angle 4 = 28^\circ + 55^\circ = 83^\circ$. Since the angle measures are equal, by the Converse of the Alternate Exterior Angles Theorem, $\ell \parallel m$. **12.** $p \parallel q$; Converse of the Corresponding Angles Theorem

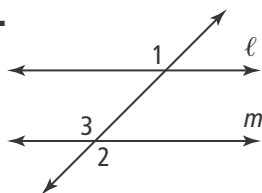


Selected Answers

Topic 2

14. $r \parallel s$; Converse of the Alternate Exterior Angles Theorem

16.



$$\angle 1 \cong \angle 2$$

Given

$$\angle 2 \cong \angle 3$$

Vert. \angle s Thm.

$$\angle 1 \cong \angle 3$$

Trans. Prop.

$$\ell \parallel m$$

Converse of the
Corr. \angle s Thm.

18. $a \perp d, b \perp d \rightarrow \angle 1 \text{ and } \angle 2 \text{ are rt. } \angle$ s

Given

Def. Rt. \angle s

$$\angle 1 \cong \angle 2$$

Def. $\cong \angle$ s

$$a \parallel b$$

Converse of the
Corr. \angle s Thm.

20. 74° ; Converse of the Alternate Exterior Angles Theorem **22. a.** 55° ; Converse of the Same-Side Interior Angles Theorem **b.** 55° ; Converse of the Corresponding Angles Theorem **24.** C

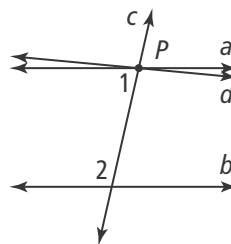
Lesson 2-3

2. Answers may vary. Sample: Chiang did not add the correct remote angles to find the exterior angles. $x = y = 125^\circ$.

4. a. Since the sum of the measures of a triangle is 180° and all three angles of an equilateral triangle have equal measure, each angle is $180^\circ \div 3 = 60^\circ$.

b. Since the sum of the measures of a triangle is 180° and the other two angles of an isosceles triangle must have the same measure, each angle is $90^\circ \div 2 = 45^\circ$. **6.** 40 **8.** 136

10. Given: line b with point P not on b



Prove: Line a is the only line through P that is parallel to line b . Construct arbitrary line c through P . Construct line a through P such that $m\angle 1 + m\angle 2 = 180^\circ$. Assume another line, d , that contains P and $a \parallel d$. By the converse of the Same-Side Interiors Angle Postulate, $a \parallel b$. By Theorem 2-8, $b \parallel d$. As both a and d contain P , they are the same line and the only line parallel to b that contains P .

12. The student should have used $w^\circ = x^\circ + 31^\circ + 44^\circ$. Then $x^\circ = 49^\circ$.

14. $x = 180^\circ - 66^\circ - 74^\circ = 40^\circ$, Alternate Interior \angle s Theorem, Triangle Angle-Sum Theorem; $y = 66^\circ$, Vert. Angles Theorem. $z = 66^\circ + 40^\circ = 106^\circ$, Triangle Exterior Angle Theorem



Selected Answers

Topic 2

16. $x = 63$ **18.** $x = 98$; $y = 38$ **20.** 139
22. 73 **24.** 27° **26.** 58° **28.** 42° , 42°
30. 144° **32.** $x = 111^\circ$, $y = 36^\circ$, $z = 76^\circ$
34. Part A $m\angle 1 = 138^\circ$; $m\angle 3 = 42^\circ$;
 $m\angle 4 = 96^\circ$; $m\angle 5 = 84^\circ$ **Part B** Yes;
 make an equiangular triangle: $m\angle 1 =$
 120° ; $m\angle 2 = m\angle 3 = m\angle 4 = 60^\circ$; $m\angle 5 =$
 120° . **Part C** Apply the Triangle
 Exterior Angle Theorems and the Linear
 Pairs Theorem: $(2 \times m\angle 5) + m\angle 2 =$
 180° ; $m\angle 5 = 2 \times m\angle 2$. Therefore,
 $m\angle 1 = 144^\circ$; $m\angle 2 = m\angle 3 = 36^\circ$;
 $m\angle 4 = 108^\circ$; $m\angle 5 = 72^\circ$.

Lesson 2-4

2. The product of the slopes is not -1 .
4. If two non-vertical lines are parallel,
 then their slopes are equal. If the slopes
 of two non-vertical lines are equal,
 then the lines are parallel. **6.** yes **8.** no
10. $y = -x + 5$ **12.** $y = -\frac{3}{16}x + 43\frac{7}{8}$
14. Vertical lines have undefined slope,
 and undefined is not a value, so it

cannot be equal to another value. It
 also cannot be multiplied by a number
 to get a value, so the product of
 undefined and another number cannot
 be -1 . **16.** They are collinear. x is a
 point on both lines. A point and a slope
 determine a line, so if the slope of both
 lines is the same, they must be the same
 line. **18.** Both slopes are 15; the lines
 are parallel. **20.** no **22.** yes **24.** no
26. $y = -4x + 23$; $y = \frac{1}{4}x - \frac{5}{2}$ **28.** Gym
 and Art Studio **30.** $(\frac{16}{19}, \frac{118}{19})$ **32.** B

Topic Review

2. same-side exterior angles **4.** remote
 interior angles **6.** alternate interior
 angles **8. a.** 42° **b.** 42° **c.** 138° **10.** 50° ,
 50° **12.** 135° **14.** 26° **16.** no; slope of
 p : $m_p = \frac{-4 - 2}{-9 - (-6)} = 2$; slope of q : $m_q =$
 $\frac{-4 - 3}{6 - 9} = \frac{7}{3}$ **18.** Vertical lines have an
 undefined slope.



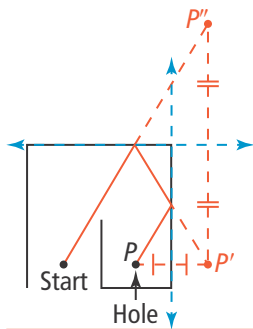
Selected Answers

Topic 3

Lesson 3-1

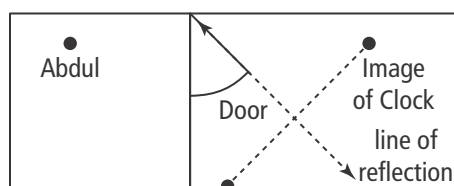
- 2.** The reflection is across the line $x = -1$. **4.** Check to see if all sides and angles are congruent. **6.** No; the side lengths appear to be different. **8.** $(1, -6)$ **10.** Students draw the point on the front wall that would reflect from Player 1 to the rear corner. **12.** Yes; squares have the same angle measures because they are right angles and the size of sides are the same length.

14.



- 16.** No; reflections change the orientation of a figure, so an even number of reflections is needed for the orientation to be the same as the original figure. **18.** No; corresponding sides do not appear to be the same size. **20.** $A'(-9, -3)$, $B'(-6, 4)$, $C'(1, -5)$ **22.** $A'(9, 5)$, $B'(6, -2)$, $C'(-1, 7)$ **24.** $A'(9, -1)$, $B'(6, -8)$, $C'(-1, 1)$ **26.** $R_f(x, y) \rightarrow (x, 10 - y)$ where f is the line $y = 5$ **28.** $R_t(x, y) = (-y, -x)$ where t is the line $y = -x$ **30.** stones 7, 8, 11, 12, 13, and 14

- 32. a.** The line of reflection contains the door.



- b.** The clock is located on the wall. **c.** No, he cannot see himself. The image of his position reflected over the line of the mirror is not in the room. **34.** C

Lesson 3-2

- 2.** The x - and y -axes are perpendicular. The lines of reflection must be parallel. **4.** $(5, -8)$ **6.** $X(6, -7)$, $Y(3, -4)$, $Z(8, -2)$ **8.** $T_{\langle 4, 4 \rangle}$ **10.** 6 units **12.** The area of the rectangle is the product of its length and width. Rigid motions preserve lengths, so the product will not change. Therefore, the area of $ABCD$ is equal to the area of $A''B''C''D''$. **14.** Answers may vary. Sample: $T_{\langle -7, 0 \rangle}$ **16.** $D'(-1, 3)$, $E'(-6, 3)$, $F'(-4, 2)$ **18.** $T_{\langle -5, 3 \rangle}$ **20.** $T_{\langle -8, 0 \rangle}$ **22.** $T_{\langle -1, -2 \rangle}$ **24.** $T_{\langle 4, 7 \rangle}$ **26.** $T_{\langle 4, 0 \rangle}$ **28.** $T_{\langle 0, 12 \rangle}$ **30.** $T_{\langle 2, -1 \rangle} \circ T_{\langle 8, 4 \rangle}$; $T_{\langle 10, 3 \rangle}$ **32.** $T_{\langle -60, 40 \rangle} \circ T_{\langle 20, 10 \rangle}$ **34.** B

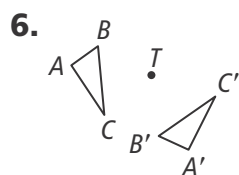
Lesson 3-3

- 2.** She did not label the image points correctly. The point that she labeled E' should be D' . The point she labeled D' should be F' . The point she labeled F' should be E' . **4.** 140° ; Line p is equidistant from each corresponding point in the original and reflected figures, so it is the angle bisector of the angles formed by each point in the figure, R , and the corresponding point in the reflected image. Likewise, q is the angle bisector of the angles formed by each point in $\angle A'B'C'$ and its reflected image. So the angles formed by each point in $\angle ABC$, R , and the corresponding point in $\angle A''B''C''$ are twice the measure of the angle formed by line p , R , and line q .

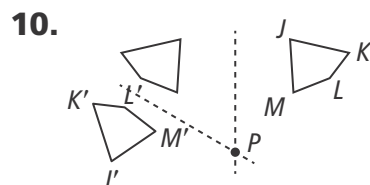


Selected Answers

Topic 3

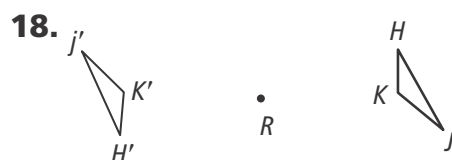
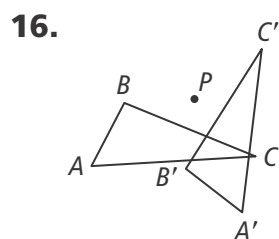


8. $X'(-3, 0)$, $Y'(4, 1)$, $Z'(-2, 5)$



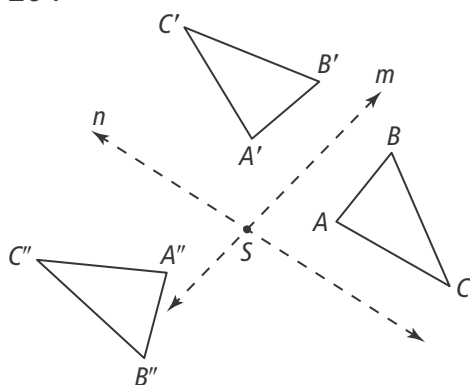
12. If it were a rotation, then Y' would be the vertex at the top of the image. $\triangle X'Y'Z'$ is a reflection of $\triangle XYZ$.

14. $A(1, 2)$, $B(-2, 4)$, $C(-6, 4)$, $D(-3, 1)$



20. $W'(-2, -4)$, $X'(3, -7)$, $Y'(11, -1)$, $Z'(6, 4)$ 22. $J'(-4, 7)$, $K'(1, 5)$, $L'(6, 1)$, $M'(3, -9)$

24. 204°



26. $(0, 6)$, $(-2.3, 5.5)$, $(-4.2, 4.2)$, $(-5.5, 2.3)$, $(-6, 0)$, $(-5.5, -2.3)$, $(-4.2, -4.2)$, $(-2.3, -5.5)$, $(0, -6)$, $(2.3, -5.5)$, $(4.2, -4.2)$, $(5.5, -2.3)$ 28. is equal to; is congruent to; not enough information; is equal to 30. **Part A** Answers may vary. Sample: Rotate Piano 1 90° counterclockwise about D , then translate 6 units left. Rotate Piano 2 180° about J and then translate 16 units left. **Part B** Rotate Piano 1 90° counterclockwise about the point 1 unit right and 3 units down from A . Rotate Piano 2 180° about the point 8 units to the left of J .

Lesson 3-4

2. Yes; a glide reflection commutes.

4. reflection 6. glide reflection

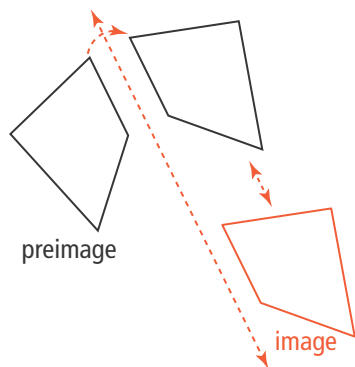
8. **Given:** $M(\triangle XYZ) \rightarrow \triangle X'Y'Z'$ and $\triangle XYZ \cong \triangle X'Y'Z'$ **Prove:** $M = R$, $R_1 \circ R_2$, or $R_1 \circ R_2 \circ R_3$ **Proof:** By Theorem 3-4, the rigid motion M is either a translation, a reflection, a rotation, or a glide reflection. (1) If M is a translation, then by Theorem 3-1 it can be expressed as $M = R_1 \circ R_2$. (2) If M is a reflection, then $M = R$. (3) If M is a rotation, then by Theorem 3-2 it can be expressed as $M = R \circ R$. (4) If M is a glide reflection, then $M = T \circ R$, which can also be written as $M = R_1 \circ R_2 \circ R_3$. Together all four cases prove $M = R$, $R_1 \circ R_2$, or $R_1 \circ R_2 \circ R_3$.



Selected Answers

Topic 3

10. Sample answer:



12. Answers may vary. Sample: Reflect across the line $y = 1$, and then translate the image left 6 units. **14.** $A'(10, 4)$, $B'(7, -8)$, $C'(-3, -9)$ **16.** $A'(1, 22)$, $B'(-2, 10)$, $C'(-12, 9)$ **18.** Answers may vary. Sample: $(T_{\langle 0, 3 \rangle} \circ R_t)(\triangle DEF) = \triangle D'E'F'$, where t is the line $x = 1$ **20.** Answers may vary. Sample: $(T_{\langle -6, 0 \rangle} \circ R_t)(\triangle DEF) = \triangle D'E'F'$, where t is the line $y = -2$ **22.** $(T_{\langle 5, 0 \rangle} \circ R_{y=6.5})(\triangle 1) = \triangle 3$; $(T_{\langle 5, 0 \rangle} \circ R_{y=6.5})(\triangle 3) = \triangle 5$; **24.** Using a glide reflection, tile 5 will reflect across a vertical line to tile 6, which should be shaped like tile 2. Tile 6 will then translate down to tile 7, and its shape should be the same as tile 6. **26.** C

Lesson 3-5

2. The horizontal line Adam drew is not a line of reflection, because the halves are not congruent. **4.** It means that when the figure is turned through an angle of 60° around a point at its center, the preimage and image are identical. **6.** 2 **8.** 180° ; the figure has point symmetry. **10.** reflectional symmetry; 1. **12.** It is possible for a figure to have reflectional but not

rotational symmetry. An example of this is a smiley face. **14.** Conditional: If a figure has 180° rotational symmetry, then it has point symmetry. Converse: If a figure has point symmetry, then it has 180° rotational symmetry. Inverse: If a figure does not have 180° rotational symmetry, then it does not have point symmetry. Contrapositive: If a figure does not have point symmetry, then it does not have 180° rotational symmetry. **16.** The figure has 180° rotational symmetry and point symmetry. **18.** reflection over a vertical line through the center **20.** 5 lines **22.** 90° , 180° , 270° **24.** Answer may vary. Sample: DID can have horizontal reflectional symmetry; MOM has vertical reflectional symmetry, 8118 can have both horizontal and vertical reflectional symmetry as well as 180° rotational and point symmetry. **26.** The structure is based on a regular hexagon. Each has six lines of reflection and rotational symmetry at multiples of 60° . **28.** A, B, D **30. Part A** Sample: square, plus sign **Part B** A figure with that rotational symmetry most likely also has horizontal and vertical reflectional symmetry. **Part C** Designs will vary, but should have the required symmetry.

Topic Review

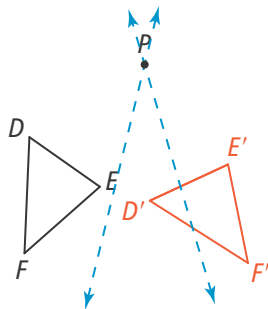
2. rotation **4.** line of symmetry **6.** preimage **8.** yes **10.** $H'(-3, -2)$, $J'(-1, 3)$, $K'(4, -3)$ **12.** $P'(-7, 5)$, $Q'(-5, 5)$, $R'(-2, -1)$ **14.** $T_{\langle -5, 2 \rangle}$ **16.** $A'(2, 2)$, $B'(2, -3)$, $C'(-3, -1)$



Selected Answers

Topic 3

18. Answers may vary. Sample:



20. $L'(6, 8)$, $M'(3, 6)$, $N'(7, -1)$ **22.** $T_{\langle 0, 5 \rangle}$
 $\circ R_m$ where m is the line $x = \frac{1}{2}$

24. reflection across the horizontal line through the center, reflection across the vertical line through the center, 180° rotation about the center

26. reflectional symmetry with lines of reflection that go through the far vertex of each purple triangle and the center of the figure; rotational symmetry about the center of the figure of 120° and 240°



Selected Answers

Topic 4

Lesson 4-1

- 2.** Multiplication changes the side lengths, so there is no composition of rigid motions that maps $ABCD$ to $EFGH$. The figures are not congruent.
- 4.** They must be the same length.
- 6.** Sample: To map a segment to a segment, first translate one endpoint onto the corresponding endpoint. Then rotate about that endpoint until the other endpoint is correctly mapped. For an angle, first translate the vertex onto the other vertex. Then rotate about the vertex until the rays are correctly mapped.
- 8.** $\triangle ABC \cong \triangle DEF$, $\triangle GHJ \cong \triangle KLM$
- 10.** Answers may vary. Sample: If $T_{(5,0)}(\triangle JKL) = \triangle RST$, then the vertices of $\triangle RST$ are $R(-1, 3)$, $S(0, 7)$, and $T(3, 2)$.
- 12.** 144 cm^2
- 14.** yes; Rotation and reflection are rigid motions, so side lengths are preserved. The sum of the side lengths is unchanged.
- 16.** yes; $(R_{y\text{-axis}} \circ R_{x\text{-axis}})(ABCD) = A'B'C'D'$
- 18.** A and C by a reflection across the vertical line halfway between; A and G by a 180° rotation about the midpoint of the segment between the centers of A and G; D and I by a translation; E and F by a translation; C and G by a glide reflection
- 20.** Sample: rotations by multiples of 45° about the center of the quiche
- 22.** C

Lesson 4-2

- 2.** The exterior of an equilateral triangle cannot have a measure of 140° . Each angle of an equilateral triangle has a measure of 60° . By the Triangle

Exterior Angle Theorem, the measure of any exterior angle must be 120° .

- 4.** Yes; each base angle has a measure of 45° .
- 6.** $m\angle A = 42^\circ$; $m\angle B = 69^\circ$

- 8.** $GH = 49$; $HJ = 35$; $JG = 35$
- 10.** 19°

- 12.** The measure of each angle in an equilateral triangle is 60° . Therefore, by the Alternate Interior Angles Theorem, $m\angle 1 = 60^\circ$. By the Same-Side Interior Angles Postulate, $\angle 1$ and $\angle 2$ are supplementary, so $m\angle 2 = 120^\circ$.

- 14.** $\triangle 2 \cong \triangle 4$ because you can reflect across their common side, and $\triangle 1 \cong \triangle 3$ because you can rotate one onto the other.
- 16.** If the segment from $(1, 5)$ to $(1, -1)$ is a leg of the triangle, then a vertex at $(-5, 5)$ will make an isosceles right triangle with leg length 6. If the segment from $(1, 5)$ to $(1, -1)$ is the base of the triangle, then a vertex at $(-2, 2)$ will make an isosceles right triangle with leg length $3\sqrt{2}$.

- 18.** $m\angle P = 51^\circ$; $m\angle Q = 78^\circ$

- 20.** $DE = 20$; $EF = 20$, $DF = 29$
- 22.** 20°

- 24.** Given: $\overline{AD} \cong \overline{BD} \cong \overline{CD}$

Answers may vary. Sample:

$\triangle ADB$ and $\triangle CDB$ are isosceles by the definition of isosceles triangles. Let $m\angle DAB = x$. $m\angle DAB = x$ by the Isosceles Triangle Theorem. $m\angle CDB = 2x$ by the Triangle Exterior Angle Theorem.

By the Triangle Angle-Sum Theorem:

$$m\angle BCD + m\angle DBC + m\angle CDB = 180$$

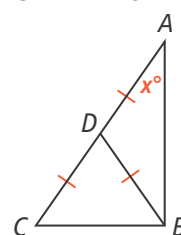
$$2 \times m\angle BCD + 2x = 180$$

$$m\angle BCD = \frac{180 - 2x}{2}$$

$$m\angle BCD = 90 - x$$

By the Triangle Angle-Sum Theorem:

$$m\angle ABC + m\angle BCA + m\angle CAB = 180$$





Selected Answers

Topic 4

$$m\angle ABC + (90 - x) + \angle BCD = 180$$

$$m\angle ABC + 90 = 180$$

$$m\angle ABC = 90$$

Therefore, $\triangle ABC$ is a right triangle because $m\angle ABC = 90$. **26.** 14.1 cm

28. 24.1 ft **30.** B

Lesson 4-3

2. $\angle F$ is not an included angle, so SAS does not apply. **4.** $\triangle JKL \cong \triangle MNP$ by SSS. $\angle JKL \cong \angle MNP$ by CPCTC. **6.** SSS

8. SAS **10.** Given $\overline{LM} \cong \overline{PN}$ and $\overline{MN} \cong \overline{LP}$. $\overline{LN} \cong \overline{LN}$ by the Reflexive Property of Congruence. $\triangle LMN \cong \triangle NPL$ by SSS.

12. Zhang is comparing the lengths of sides within the same triangle. SAS requires comparing the lengths of two sides in one triangle to the lengths of two sides in a second triangle.

14. Given $\overline{KL} \cong \overline{NL}$ and $\overline{JL} \cong \overline{ML}$, by the Vertical Angles Theorem $\angle KLJ \cong \angle MLN$. $\triangle JKL \cong \triangle MNL$ by SAS.

16. First show that the two triangles formed by the diagonals of each triangle are congruent by SSS. Then all corresponding parts of those triangles are congruent by CPCTC. Angles A and C can be shown to be congruent to angles N and L by the Angle Addition Postulate. **18.** $m\angle RTS = f^\circ$; Given $\overline{DE} \cong \overline{SR}$, $\overline{DF} \cong \overline{ST}$, and $\angle EDF \cong \angle SRT$. $\triangle DEF \cong \triangle RST$ by SAS. $\angle DFE \cong \angle RTS$ by CPCTC. $m\angle DEF = f^\circ$, so $m\angle RTS = f^\circ$.

20. To show congruency by SSS, you need to know $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$.

22. Yes; $\overline{PQ} \cong \overline{PS}$, $\overline{PR} \cong \overline{PR}$, and $\angle QPR \cong \angle SPR$, so $\triangle PQR \cong \triangle PSR$ by SAS.

24. 1,700 ft; The two triangles are

congruent by SAS. The distance traveled is the sum of the perimeters of the triangles. The triangles are congruent by SAS, so the distance is twice the perimeter of one triangle. All of the side lengths of the triangle on the left are given so the distance traveled is $2(350 + 300 + 200) = 1,700$.

26.

	Yes	No
$\overline{FG} \cong \overline{JK}$, $\overline{GH} \cong \overline{KL}$, $\overline{FH} \cong \overline{JL}$	X	
$\overline{FG} \cong \overline{JK}$, $\overline{FH} \cong \overline{JL}$, $\angle FHG \cong \angle JLK$		X
$\overline{GH} \cong \overline{KL}$, $\overline{FG} \cong \overline{JK}$, $\angle FGH \cong \angle JKL$	X	
$\overline{GH} \cong \overline{KL}$, $\overline{FH} \cong \overline{JL}$, $\angle FHG \cong \angle JLK$	X	

28. Part A The paths are congruent triangles, by SAS. **Part B** No; while the triangles are congruent, the angles at A and B are not corresponding angles. The triangles are not isosceles so the angles cannot have the same measure.

Lesson 4-4

2. The triangles have only one pair of corresponding congruent sides ($\overline{AB} \cong \overline{CD}$) and no pairs of corresponding congruent angles.

4. No; there are only two pairs of corresponding parts given. **6.** $x = 4$

8. congruent by ASA **10.** $\overline{CX} \cong \overline{CX}$ by the Reflexive Property of Congruence. So there are two pairs of angles and the included sides that are congruent. Stacy could use ASA.

12. Given: $\angle BAC \cong \angle DEC$,
 $\angle ABC \cong \angle EDC$, $\overline{BC} \cong \overline{DC}$

Prove: $\angle CAE \cong \angle CEA$



Selected Answers

Topic 4

Statement	Reasons
$\angle BAC \cong \angle DEC$, $\angle ABC \cong \angle EDC$, $\overline{BC} \cong \overline{DC}$	Given
$\triangle ABC \cong \triangle EDC$	AAS
$\overline{AC} \cong \overline{EC}$	CPCTC
$\triangle ACE$ is isosceles	Definition of isosceles
$\angle CAE \cong \angle CEA$	Isosceles Triangle Theorem

14. Answers may vary. Sample: Translate point D onto point J , then rotate $\triangle DEF$ about point J such that point E lies on point K . After the rigid motions, point D lies on point J and point E lies on point K but point F does not lie on point L , so SSA is not sufficient to prove triangles congruent.

16. Given: $\angle P \cong \angle S$, $\angle Q \cong \angle T$, $\overline{QR} \cong \overline{TU}$
Prove: $\triangle PQR \cong \triangle STU$

Statement	Reasons
$\angle P \cong \angle S$, $\angle Q \cong \angle T$, $\overline{QR} \cong \overline{TU}$	Given
$m\angle R = 180 - m\angle P - m\angle Q$ $m\angle U = 180 - m\angle S - m\angle T$	Triangle Angle-Sum Theorem
$m\angle U = 180 - m\angle P - m\angle Q$	Substitution
$\angle R \cong \angle U$	Transitive Property of Equality
$\triangle PQR \cong \triangle STU$	ASA

18. Yes; $\triangle ADC \cong \triangle GJK$ by AAS, $\overline{AC} \cong \overline{GJ}$ by CPCTC, $\triangle ABC \cong \triangle GHJ$ by SAS. Since $ABCD$ and $GHJK$ are composed of corresponding congruent triangles, $ABCD \cong GHJK$. **20.** Given $\overline{LM} \cong \overline{ON}$. $\angle L \cong \angle N$ and $\angle M \cong \angle O$ because they are alternate interior

angles. $\triangle LMP \cong \triangle NOP$ by ASA. The heights of congruent triangles are congruent, so P is halfway up the cliff.

22. Yes; the wall would make right angles where it bisects \overline{PO} , so there would then be two quadrilaterals with all sides and angles congruent. **24.** C

Lesson 4-5

2. The HL Theorem can only be applied to right triangles. The triangles in the figure have congruent angles, but they are not necessarily right angles. **4.** For any acute or obtuse triangle, there are two possible triangles that can have two congruent sides and a nonincluded angle. For a right triangle, there is only one possible triangle that has two congruent sides and a nonincluded angle. **6.** $\overline{TU} \cong \overline{UV}$ **8.** $PS = 15$ **10.** Use the Pythagorean Theorem to show $GJ = 13$. Use the Exterior Angle Theorem to show $m\angle KLM = 90^\circ$.

12. An angle bisector of any angle of an equilateral triangle bisects the opposite side, forming right triangles. The two right triangles formed have congruent hypotenuses and congruent legs. For an isosceles triangle, only the angle bisector of the angle between the congruent sides bisects the opposite side and forms right angles.

14. $\overline{BC} \cong \overline{DF}$ or $\overline{AC} \cong \overline{EF}$

16. $\overline{AC} \cong \overline{EF}$ or $\overline{AB} \cong \overline{ED}$

18. $DE = BC$ and $EF = 12$ or $AB = 33$

20. Given that $\overline{EF} \cong \overline{GH}$ and G is the midpoint of \overline{EJ} , by the definition of midpoint, $\overline{EG} \cong \overline{GJ}$. Since $\angle EGF$ and $\angle GJH$ are right angles, by the HL Theorem, $\triangle EFG \cong \triangle GHJ$.



Selected Answers

Topic 4

22. a. The HL Theorem only applies to right triangles. The figure does not label $\angle B$ or $\angle D$ as right angles. Also, there is no information given about the hypotenuses of the triangles.

b. No; the only information given is the congruency of two sides. You need at least one other angle or one other side to prove congruency.

- 24.** I. $\overline{PQ} \cong \overline{ST}$ and $\overline{QU} \cong \overline{TR}$ A. ASA
 II. $\overline{PU} \cong \overline{SR}$ and $\overline{QU} \cong \overline{TR}$ B. AAS
 III. $\overline{QU} \cong \overline{TR}$ and $\angle U \cong \angle R$ C. SAS
 IV. $\overline{QU} \cong \overline{TR}$ and $\angle P \cong \angle S$ D. HL

26. Part A AG and CG **Part B** Yes; she can use the Pythagorean Theorem to show that $KJ = DC$, and then apply the HL Theorem. **Part C** Answers may vary. Sample: She can measure to show that $GB = CH$, and then apply the HL Theorem to prove that $\triangle GBC \cong \triangle CHG$.

Lesson 4-6

2. Since $\overline{AC} \cong \overline{EC}$, $\angle C \cong \angle C$, and $\angle CDA \cong \angle CBE$, by AAS $\triangle ACD \cong \triangle ECB$.

4. $\triangle KJM$, $\triangle LMJ$, $\triangle MLK$; Sample:

Each triangle is a right triangle with a diagonal as the hypotenuse and a corner of the rectangle as the right angle. **6.** $\angle BAD$ **8.** $\angle FHG$ **10.** SAS

12. Given: $\angle CHB$ and $\angle EHF$ are right angles, $\overline{CB} \cong \overline{EF}$, $\angle EBF \cong \angle CFB$, and $\angle DAF \cong \angle DGB$ **Prove:** $\overline{AF} \cong \overline{GB}$

Proof: Since $\angle HBF \cong \angle HFB$, $\triangle HFB$ is isosceles and therefore $\overline{HB} \cong \overline{HF}$. $\angle CHB$ and $\angle EHF$ are right angles and $\overline{CB} \cong \overline{EF}$, so $\triangle CHB \cong \triangle EHF$ by HL. By CPCTC, $\angle BCH \cong \angle FEH$ and $\overline{CH} \cong \overline{EH}$. So, by the Segment Addition Postulate and substitution, $\overline{BE} \cong \overline{FC}$. Since it is also given that $\angle CAF \cong \angle EGB$, by AAS,

$\triangle CFA \cong \triangle EBG$. Therefore, by CPCTC, $\overline{AF} \cong \overline{GB}$.

14. The angles that Dyani says are congruent are not angles in the congruent triangles, so she cannot use CPCTC to show that they are congruent.

16. Corresponding sides: \overline{CA} and \overline{DA} , \overline{AE} and \overline{AB} , \overline{EC} and \overline{BD} ; Corresponding angles:

$\angle CAE$ and $\angle DAB$, $\angle AEC$ and $\angle ABD$, $\angle ECA$ and $\angle BDA$ **18.** \overline{GK} **20.** \overline{WY}

22. $\angle AEF \cong \angle ADF$ since all right angles are congruent. It is given that $\angle EAF \cong \angle DAF$ and $\overline{AF} \cong \overline{AF}$ by Refl. Prop.

of Congruence, so $\triangle AEF \cong \triangle ADF$ by AAS. $\overline{EF} \cong \overline{DF}$ by CPCTC, $\angle CEF \cong \angle BDF$ since all right angles are congruent and $\angle EFC \cong \angle DFB$ by the Vertical Angles Thm. $\triangle CEF \cong \triangle BDF$ using ASA. Since $\overline{EC} \cong \overline{DB}$ by CPCTC and $\overline{BC} \cong \overline{CB}$ by Refl.

Prop. of Congruence, $\triangle BCE \cong \triangle CBD$ by the HL Theorem. **24.** Not necessarily; the only information given in the diagram is that the sides are congruent. Two sets of congruent sides is not sufficient to prove $\triangle ABC$ and $\triangle DBC$ are congruent. **26.** 9.92 m **28** B

Topic Review

2. congruent **4.** congruence

transformation **6.** Yes; map one figure to the other by reflecting it across a vertical line between the two figures.

8. Yes; it will map the figure onto itself.

10. 70° , 70° **12.** SSS; Two pairs of corresponding sides are given and the third pair is congruent by the Reflexive Property of Congruence.

14. There are 28 pairs from the smaller triangles. There are 6 pairs of from triangles bordered by two sides of the table and a diagonal. There are 2 pairs of triangles



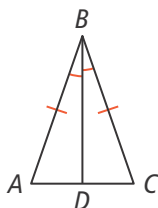
Selected Answers

Topic 4

that are formed by the one side of the table and two diagonals. So, there are a total of 36 congruent triangles.

16. AAS **18.** There is not enough information to tell whether the triangles are congruent.

20.



$\angle ABD \cong \angle CBD$ by definition of angle bisector. $\overline{AB} \cong \overline{CB}$ by definition of

isosceles triangle. $\overline{BD} \cong \overline{BD}$ by the Reflexive Property of Congruence. $\triangle ABD \cong \triangle CBD$ by SAS. **22.** $\angle D \cong \angle C$ and $\overline{ED} \cong \overline{BC}$ are given. By the Vertical Angles Theorem, $\angle EGD \cong \angle BGC$, so $\triangle DEG \cong \triangle CBG$ by AAS. $\overline{EG} \cong \overline{BG}$ and $\overline{DG} \cong \overline{CG}$ are congruent using CPCTC. $\overline{AE} \cong \overline{FB}$ is given. Apply the Segment Addition Postulate to get $DG + GB + BF = DF$ and $CG + GE + EA = CA$. Using the Substitution Property and the Transitive Property, $DF = CA$, so $\overline{DF} \cong \overline{CA}$. $\triangle ABC \cong \triangle FED$ by SAS.



Selected Answers

Topic 5

Lesson 5-1

2. Answers may vary. Sample: If a point is equidistant from two sides of an angle, the lengths of the segments which connect the point to the sides and are perpendicular to the sides are equal. **4.** The four sides of quadrilateral $AXBY$ are congruent. Sample answer: All four sides have equal length because $AX = XB$, $XB = BY$, and $BY = YA$ by the Perpendicular Bisector Theorem.

6. 4 8. $(2x - 4)^2$ **10. Given:** $BD = CD$, $\overline{BD} \perp \overline{AB}$, $\overline{CD} \perp \overline{AC}$ **Prove:** $\angle BAD \cong \angle CAD$
It is given that $BD = CD$, so by the definition of congruent segments, $\overline{BD} \cong \overline{CD}$. By the Reflexive Property, $\overline{AD} \cong \overline{AD}$. It is given that $\overline{BD} \perp \overline{AB}$ and $\overline{CD} \perp \overline{AC}$, so by the definition of perpendicular, $\angle ABD$ and $\angle ACD$ are right angles. By the definition of right triangles, $\triangle ACD$ and $\triangle ABD$ are right triangles. So, by HL, $\triangle ACD \cong \triangle ABD$. Therefore, by CPCTC, $\angle BAD \cong \angle CAD$. **12.** Sample: It is not true that $BC = CD$. To use the Angle Bisector Theorem, \overline{CD} must be perpendicular to \overline{ED} . **14. 22 16. 21 18. 42° 20. 31 22. 52 ft 24. 849 ft² 26. B**

Lesson 5-2

2. The circle does not contain all three vertices, so it is not the circumscribed circle. It appears that he used the midpoint of \overline{XZ} as the center instead of the circumcenter. **4.** No; the inscribed circle is inside the triangle and intersects the sides, and the circumscribed circle is outside the triangle and intersects the vertices. **6. 5**

8. 30 10. Given: $\triangle ABC$ with angle bisector q of $\angle A$, angle bisector r of $\angle B$, and angle bisector s of $\angle C$. **Prove:** q , r , and s are concurrent at point P equidistant from the sides of $\triangle ABC$.

Statement	Reason
1) q is the angle bisector of $\angle A$, and r is the angle bisector of $\angle B$.	1) Given
2) P is the point of intersection of q and r .	2) q and r are not parallel so they intersect.
3) Construct \overline{MP} such that $\overline{AB} \perp \overline{MP}$ and M is on \overline{AB} and \overline{OP} such that $\overline{AC} \perp \overline{OP}$ and O is on \overline{AC} .	3) There exists a unique line perpendicular to a line, containing a point not on the line.
4) $MP = OP$	4) Angle Bisector Theorem
5) Construct \overline{NP} such that $\overline{BC} \perp \overline{NP}$ and N is on \overline{BC} .	5) There exists a unique line perpendicular to a line, containing a point not on the line.
6) $NP = OP$	6) Angle Bisector Theorem
7) $MP = NP$	7) Transitive Property of Equality
8) $\angle BCP \cong \angle ACP$	8) Converse of the Angle Bis. Thm.

continued...



Selected Answers

Topic 5

Statement	Reason
9) P is on s	9) Definition of angle bisector
10) q , r , and s are concurrent at P .	10) Definition of concurrent
11) P is equidistant from \overline{AB} , \overline{BC} , and \overline{AC} .	11) Definition of equidistant

12. $DV \neq DT$, and $DM \neq DV$. $DT = TE = 33.5$. $DM = MF = 38$. Perimeter is $33.5 + 17 + 38 = 88.5$. **14.** 1; The lengths of the segments formed by the perpendicular to the hypotenuse are $3 - r$ and $4 - r$, so $(3 - r) + (4 - r) = 5$, or $r = 1$. **16.** T **18.** 8.2 **20.** 9 **22.** 6.9 **24.** 21.0 m^2 ; the center of the circle is the intersection of the perpendicular bisectors. The radius is $\sqrt{1.8^2 + 2.4^2} = 3$. The area of the patio is $\pi(3)^2 \approx 28.27$. The area covered by the sun shade is $(0.5)(2.5)(3.1 + 2.7) = 7.25$, so the area not covered is about $28.27 - 7.25 = 21.02 \approx 21.0 \text{ m}^2$. **26.** B, C **28. Part A** and **Part B** Check students' work. **Part C** They should use an equilateral triangle. The inscribed circle in an equilateral triangle covers the largest portion of the triangle.

Lesson 5-3

2. The orthocenter is the concurrency of the lines containing the altitudes, which are upright or right, compared to the side to which they are drawn. **4.** The orthocenter is outside an obtuse triangle because the altitude from the vertices of the acute angles to the line containing the opposite side is outside the triangle. A centroid is always

inside a triangle because the medians are always inside the triangle. **6.** The orthocenter is the vertex of the right angle. **8.** $(\frac{1}{2}, 4)$ **10.** Find the slopes of two sides. Find the slopes of the lines perpendicular to the sides, and then write equations for the perpendicular lines through the opposite vertices. Solve the system of equation to find the coordinates of the orthocenter.

12. The perpendicular segments shown are not altitudes. Altitudes are perpendicular segments that have an endpoint at a vertex. The Concurrency of Altitudes Theorem cannot be applied. The point of concurrency shown is the circumcenter. **14.** $(2, \frac{8}{3})$ **16.** 22.5

18. a. on the triangle; right triangle
b. inside the triangle; acute triangle
c. inside the triangle; acute triangle
d. outside the triangle; obtuse triangle
20. 2 ft **22.** Answers may vary. Sample: The designer drew the medians of the triangle, so the point of concurrency shown is the centroid. An altitude is the shortest segment from a vertex to the opposite side, so the point of concurrency should be the orthocenter. **24.** D

Lesson 5-4

2. The triangle has three different angle measures, so none of the angles are congruent. **4.** Use the fact that the angles of a triangle sum to 180° to find the measure of the third angle. Then, by comparing that angle measure to 50° , determine which side length is greater. **6.** \overline{PR} **8.** no **10.** yes
12. between 7 in. and 37 in.
14. Answers may vary. Sample: \overline{BD} is



Selected Answers

Topic 5

an altitude of $\triangle ABC$. The shortest distance from a point to a line is the line perpendicular to that line through the point. Therefore, \overline{AD} is the shortest line that can be drawn from A to \overline{BD} , so $AB > AD$. By the same argument, \overline{CD} is the shortest line that can be drawn from C to \overline{BD} , so $BC > DC$. Adding the left sides and the right sides of those inequalities results in $AB + BC > AD + DC$. Using the Segment Addition Postulate, $AD + DC = AC$. Therefore, by substitution, $AB + CB > AC$.

16. Answers may vary. Sample: Tia's error was not looking at the units. There are 100 cm in a meter, so $1.7 \text{ m} = 170 \text{ cm}$. The largest angle is R .

18. $\angle H$ **20.** \overline{OP} **22.** yes **24.** no

26. between 2 in. and 22 in.

28. between 100 m and 500 m

30. Fisher Rd. and King Ln. **32.** \overline{SR} , \overline{QR} ,

\overline{SQ} **34.** $7 < x < 33$ **36. Part A** \overline{NP} , \overline{MP} ,

\overline{MN} **Part B** $\angle R$, $\angle Q$, $\angle S$; Sample: $\triangle QRS$ is a right triangle with $QS = NP = 6 \text{ ft}$ and $QR = 10 \text{ ft}$, so $SR = 8 \text{ ft}$ by the Pythagorean Theorem. So $QS < RS < QR$. So, by Theorem 5-9, $m\angle R < m\angle Q < m\angle S$.

Part C No. Since Ramp B cannot be steeper than 45° , $m\angle R \leq 45^\circ$. So there are two cases. Case 1: If $m\angle R < 45^\circ$, then $\angle R$ must be the smallest angle in $\triangle QRS$. So, by Theorem 5-10, the side opposite $\angle R$, \overline{QS} , must be the shortest side. $QS = 6 \text{ ft}$, so $SR > 6 \text{ ft}$. Case 2: If $m\angle R = 45^\circ$, then $m\angle R = m\angle Q = 45^\circ$, and $QS = SR = 6 \text{ ft}$. In either case, SR cannot be less than 6 ft.

Lesson 5-5

2. No; the Converse of the Hinge Theorem cannot be applied because

it requires that the two triangles have two pairs of congruent sides. **4.** Both require two sides of one triangle to be congruent to two sides of another triangle. The included angles are not congruent for the Hinge Theorem, but they are congruent for SAS. **6.** $m\angle SQT < m\angle QSR < m\angle PTU$ **8.** Two side lengths of one triangle must be congruent to those of another triangle in order for the Hinge Theorem to be applied. The scissors shown have blades that are congruent to each other, not to those of the triangle formed by the other pair of scissors. **10. a.** $x > 6$ **b.** $x < 7$ **12. a.** $x > 12$ **b.** $x < 11$ **14.** 7:00; when a clock starts at 12:00, the hands are together so the distance between the tips is the smallest. Even though the hour is also moving, it is moving much slower than the minute hand so the distance between the tips increases until the hands are opposite each other, and then the distance decreases.

16. No; you can only determine that d increases as the angle increases. **18.** C

Topic Review

2. incenter **4.** orthocenter **6.** equidistant

8. 40 **10.** 25° **12.** 9.8 cm **14.** A **16.** 5

18. \overline{WF} **20.** (8, 6) **22.** 90° **24.** no

26. yes **28.** $\angle Q$ **30.** If the third side were smaller than the sum of the other two sides, the triangle would not close. If it were equal to the sum of the other two sides, the two sides would form the third side. **32.** $x < 8$ **34.** Cameron cannot apply the Converse of the Hinge Theorem because the diagram does not show that the triangles have two pairs of corresponding congruent sides.



Selected Answers

Topic 6

Lesson 6-1

2. The formula $180^\circ \cdot (n - 2)$ gives the sum of the interior angles, not the exterior angles. The sum of the exterior angles of a 25-gon is 360° . **4.** 49; Any convex n -gon can be decomposed into $n - 2$ triangles. **6.** 360° **8.** 80° **10.** 170° **12.** 72° , 90° , 18° **14.** Jayesh should have multiplied 180° by 7 and then divided by 9. **16.** The base angles of each triangle in the star are each 72° since they are the exterior angles of a regular pentagon. Since the 2 base angles of each triangle are the same, they are each isosceles. All of the triangles each have a side length in common, the side between the two base angles. So, they are all congruent by ASA. **18.** 900° ; 128.6° **20.** 5° **22.** $x = 70$; 70° , 84° , 116° , 90° **24.** $x = 40$; four 100° interior angles and two 160° interior angles **26.** She can use the equilateral triangles, squares, and regular hexagons. The sides of the figure must be able to align with each other without overlapping. So the interior angles will form 360° around a point. The interior angles of each polygon are 60° , 90° , 108° , and 120° . The triangle, square, and hexagon have an interior angle that divides evenly into 360° .

- 28.** I. 4 — A. 120°
 II. 6 — B. 157.5°
 III. 16 — C. 160°
 VI. 18 — D. 90°

30. Part A Yes; When the tables are put together, the interior angle

measure where each table meets is 120° . If a polygon is formed by the tables, it will be regular. Solve $\frac{180^\circ \cdot (n - 2)}{n} = 120^\circ$ to get $n = 6$, which is a whole number, so a regular hexagon can be formed from placing the tables together.

Part B 6 **Part C** 54° and 126°

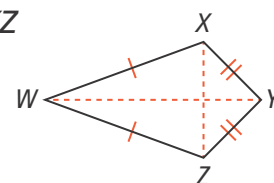
Lesson 6-2

2. According to Theorem 6-5, the diagonals of an isosceles trapezoid are congruent. However, the legs of $PQRS$ are not congruent, so it is not an isosceles trapezoid. **4.** Yes; the diagonals of every kite are perpendicular, so the intersection of the diagonals forms 4 right angles. Each side of the kite is a hypotenuse of a right triangle, with one segment of each diagonal as the legs.

6. 34° **8.** 16 ft **10.** 103°

12. Given: Kite $WXYZ$

Prove: $\overline{WY} \perp \overline{XZ}$



Statements	Reasons
1) $WX = WZ$ and $YX = YZ$	1) Given
2) W and Y are on the perpendicular bisector of \overline{XZ}	2) Converse of Perpendicular Bisector Thm.
3) \overline{WY} is the perpendicular bisector of \overline{XZ}	3) Two points define a unique line
4) $\overline{WY} \perp \overline{XZ}$	4) Def. of perpendicular bisector



Selected Answers

Topic 6

14. Points B and D are not equidistant from A and C , so \overline{BD} is not the perpendicular bisector of \overline{AC} . **16.** 9.2 m

18. Given: Isosceles trapezoid $ABCD$

Prove: $\overline{AC} \cong \overline{DB}$

Statements	Reasons
1) $\overline{AB} \cong \overline{DC}$	1) Given
2) $\angle DAB \cong \angle ADC$	2) Thm. 6-4
3) $\overline{AD} \cong \overline{AD}$	3) Refl. Prop. of \cong
4) $\triangle DAB \cong \triangle ADC$	4) SAS
5) $\overline{AC} \cong \overline{DB}$	5) CPCTC

20. 30 ft **22.** 8 times **24.** diagonals; perpendicular **26. Part A** 6 ft, 8 ft, 10 ft; The new middle board is the midsegment of the trapezoid formed by existing boards, so it will have a length of 8 ft at its midsegment. The other two boards will be midsegments of trapezoids formed by the top existing board and the new 8 ft board, and the bottom existing board and the new 8 ft board. They will have lengths at their midsegments of 6 and 10 ft. **Part B** The difference between the bases of the new boards will be the same as the difference between the bases of the bottom board.

Part C Cindy should find the thickness of the boards so she knows the size of nails needed to go through the boards into the triangular supports.

Lesson 6-3

2. The diagonals of a parallelogram bisect each other, but they are not necessarily congruent. **4.** $m\angle J$ or $m\angle L$;

Answers may vary. Sample: $\angle J$ and $\angle L$ are the angles consecutive to $\angle K$. So, $m\angle K + m\angle J = 180^\circ$ and $m\angle K + m\angle L = 180^\circ$. Since $m\angle K = (3x + 8)^\circ$, $(3x + 8)^\circ + m\angle J = 180^\circ$ and $(3x + 8)^\circ + m\angle L = 180^\circ$. Which means that $180^\circ - (3x + 8)^\circ = m\angle J$ and $180^\circ - (3x + 8)^\circ = m\angle L$.

6. 10 **8.** 81° **10.** 8 **12.** 124° **14.** The student did not observe that adjacent sides are congruent and opposite sides are not congruent. The shape is a kite.

16. No; not all angles are congruent. Check students' work. **18.** $AB = 13$, $DE = 5$ **20.** 16 **22.** 20 **24.** 10 inches; Because the diagonals bisect each other, the height of the crank is half the height of the platform. **26.** Yes; each parallelogram abuts each square with consecutive sides, and opposite sides are equal, so corresponding sides of the parallelograms are congruent. At the point where all figures meet, let the measure of the vertex angle of one parallelogram be a and let the measure of the vertex angle of the other parallelogram be b . The measures of the vertex angles of the squares are both 90° . The angles around a point must add up to 360° , so $a + 90^\circ + b + 90^\circ = 360^\circ$ and $a + b = 180^\circ$. So, the angles must be supplementary. Consecutive angles in parallelograms must also be supplementary, and opposite angles are congruent. So, the corresponding angles between two parallelograms are also congruent. Because all corresponding sides and angles are congruent, the parallelograms are congruent. **28.** B



Selected Answers

Topic 6

Lesson 6-4

2. Rochelle does not have sufficient information to conclude that $ABCD$ is a parallelogram. It may be an isosceles trapezoid. **4.** 115° **6.** 10 **8.** Yes; \overline{AB} and \overline{DC} are both congruent and parallel.

10. You could use a ruler to measure all four sides. If both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram by Theorem 6-11. You could use a protractor to measure all four interior angles. If both pairs of opposite angles are congruent, then the quadrilateral is a parallelogram by Theorem 6-13.

12. By the Converse of the Same-Side Interior Angles Postulate, $\overline{FJ} \parallel \overline{GH}$ and $\overline{FG} \parallel \overline{JH}$. Quadrilateral $FGHJ$ is a parallelogram by definition.

14. $m\angle L + m\angle M + m\angle N + m\angle O = 360^\circ$, because the sum of the interior angle measures of a quadrilateral is 360° . Substitute $m\angle L$ for $m\angle N$ and $m\angle M$ for $m\angle O$, so $2(m\angle L + m\angle M) = 360^\circ$, or $m\angle L + m\angle M = 180^\circ$. Similarly, substitute $m\angle L$ for $m\angle N$ and $m\angle O$ for $m\angle M$ in the original equation, so $m\angle L + m\angle O = 180^\circ$. Applying Theorem 6-12, quadrilateral $LMNO$ is a parallelogram.

16. a. Yes; opposite sides of the quadrilateral are congruent, so the figure is a parallelogram by Theorem 6-11. **b.** No; $52^\circ + 118^\circ = 170^\circ$, so consecutive angles are not supplementary. The figure cannot be a parallelogram. **18.** $w = 4$, $z = 5$

20. Given $\overline{KL} \parallel \overline{JM}$ and \overline{JL} is a transversal intersecting the parallel lines, $\angle KLJ$ and $\angle MJL$ are alternate interior angles, which are congruent by the Alt. Int. Angles Thm. Given \overline{KL}

$\cong \overline{JM}$ and $\overline{JL} \cong \overline{JL}$ by the Refl. Prop. of Congruence, so $\triangle KLJ \cong \triangle MJL$ by SAS. By CPCTC, $\overline{JK} \cong \overline{LM}$. Since both pairs of opposite sides are congruent, $JKLM$ is a parallelogram by Theorem 6-11.

22. Yes; because he marks every $\frac{1}{2}$ inch along the parallel lines, the opposite $\frac{1}{2}$ -inch sides are parallel and congruent, so the figure with the $\frac{1}{2}$ -inch sides and the sides made by the icing connecting the dots is a parallelogram. So, the lines drawn by connecting the dots are parallel to each other. **24.** Possible answers: (0, 6), (6, 2), or $(-4, -8)$ **26. Part A** Yes; the top is congruent to the bottom, and the sides are congruent. A quadrilateral with two pairs of congruent opposite sides is a parallelogram. **Part B** The sides remain 24 inches long.

Part C If diagonals were added, the gate would no longer be able to swing up and store out of the way, because the triangles formed would be rigid.

Lesson 6-5

2. The diagonals of a rectangle are not perpendicular bisectors, so $\angle RPS$, $\angle SPT$, $\angle TPQ$, and $\angle QPR$ are not right angles.

4. 10 **6.** 5.0 **8.** 9.2 **10.** 45° **12.** 11

14. The diagonals of a rhombus bisect each other, but they are not congruent. So, $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$.

16. Given: Parallelogram $ABCD$ is a rhombus **Prove:** $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$, $\angle 5 \cong \angle 6$, $\angle 7 \cong \angle 8$

Proof: By the definition of rhombus, $\overline{AB} \cong \overline{CB} \cong \overline{CD} \cong \overline{AD}$. Since $ABCD$ is a parallelogram and the diagonals of a parallelogram bisect each other,



Selected Answers

Topic 6

$\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$. By the Reflexive Property, $\overline{AE} \cong \overline{AE}$, $\overline{BE} \cong \overline{BE}$, $\overline{CE} \cong \overline{CE}$, and $\overline{DE} \cong \overline{DE}$. So, by SSS, $\triangle AEB \cong \triangle CEB \cong \triangle AED \cong \triangle CED$. Therefore, by CPCTC, $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$, $\angle 5 \cong \angle 6$, and $\angle 7 \cong \angle 8$.

18. 57° **20.** 90° **22.** 10 **24.** 38°

26. 12.2 **28.** 2.8 **30.** 9.7 **32.** No; if he sets up the pen so one diagonal is 10 ft, the other diagonal will be about 6.6 ft. So, the total area will be about 35 ft^2 .

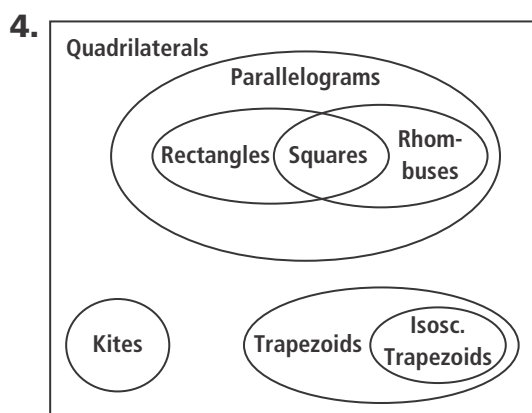
34. $m\angle 1 = m\angle 3 = 40^\circ$, $m\angle 2 = m\angle 4 = 52^\circ$; The support consists of 3 stacked rectangles. The angles are formed by the diagonals of these rectangles.

$m\angle 1 = m\angle 3 = 17^\circ + 23^\circ = 40^\circ$.

$m\angle 2 = m\angle 4 = 23^\circ + 29^\circ = 52^\circ$. **36.** C

Lesson 6-6

2. There is no indication that $DEFG$ is a parallelogram. A figure must be a parallelogram in order to apply the theorems about diagonals.



6. square **8.** rectangle **10.** rhombus; By the Alternate Interior Angles Theorem and the Triangle Angle-Sum Theorem, the vertical angles at U are right angles.

12. Becky is incorrect because the figure must be a parallelogram in order

to classify it as a rhombus. The opposite angles are not congruent, so the figure is not a parallelogram.

14. Given: $ABCD$ is a parallelogram, $\angle ABE \cong \angle BAE$. **Prove:** $ABCD$ is a rectangle. $\overline{BE} \cong \overline{AE}$ by the Converse of the Isosceles Triangle Theorem. Because $ABCD$ is a parallelogram, its diagonals bisect each other, and therefore $\overline{DE} \cong \overline{BE}$ and $\overline{CE} \cong \overline{AE}$. By the Transitive Property, $\overline{BE} \cong \overline{DE} \cong \overline{AE} \cong \overline{CE}$. By the Segment Addition Postulate and definition of congruent segments, $\overline{BD} \cong \overline{CA}$. By Theorem 6-21, $ABCD$ is a rectangle. **16.** $2x\sqrt{10}$ **18.** No; the segment shown to be perpendicular to one of the diagonals is not a diagonal.

20. square **22.** $x = 7$ **24.** $x = 18$

26. 492 in.; The figure at the bottom has congruent perpendicular diagonals that bisect each other, so it is a square with a side length of 40 in. The middle two figures have perpendicular diagonals and opposite congruent sides, so they are rhombuses with side lengths of 30 in. The top figure has a diagonal that bisects angles, so it is a rhombus with a side length of 23 in.

28.

	Yes	No
Square	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Rhombus	<input checked="" type="checkbox"/>	<input type="checkbox"/>
Parallelogram	<input checked="" type="checkbox"/>	<input type="checkbox"/>
Rectangle	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Trapezoid	<input type="checkbox"/>	<input checked="" type="checkbox"/>



Selected Answers

Topic 6

30. Part A Sample answer:

Shape	Diagram
Square	
Kite	
Rectangle	
Quadrilateral	

Part B Yes; a rhombus that is not a square is not possible because the diagonals must bisect each other, but they cannot be congruent. A parallelogram that is not a rectangle is not possible because the diagonals are congruent. **Part C** square; The area of the square would be $2(0.5)(2.5)(5) = 12.5 \text{ cm}^2$. The area of the rectangle depends on the angle at which the diagonals meet. For example, the side lengths could be 3 and 4, in which case the area would be 12 cm^2 .

Topic Review

2. square 4. interior 6. 540° ; 360°
 8. yes; 90° , 135° , 135° , 90° , 135° , 135°
 10. 7 cm 12. 85° 14. He confused the lengths of the north and west sides; opposite sides of a parallelogram are congruent. 16. 48 18. 6 20. No; the quadrilateral could be a rhombus that is not a square. 22. rhombus 24. No; the diagonals of both a kite and a rhombus are perpendicular, but if they bisect each other, the figure is a parallelogram, and therefore is a rhombus.



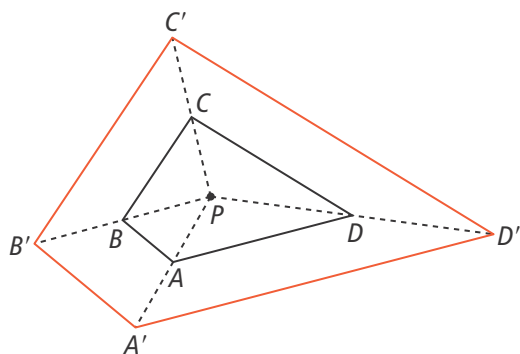
Selected Answers

Topic 7

Lesson 7-1

2. Emilia added 2 to each coordinate of the preimage. She should have multiplied each coordinate by 2.
4. The vertices of the image are the same as the vertices of the preimage. To find the vertices of the image, multiply each coordinate by 1. Multiplying a number by 1 does not change it. **6.** $\frac{1}{2}$ **8.** $A'(0, 8)$, $B'(0, 2)$, $C'(-9, 2)$ **10.** She computed the ratio of two side lengths from the preimage instead of the ratio of a side of the image to the corresponding side of the preimage. **12.** Translate by $\langle -a, -b \rangle$, then dilate with center at the origin by scale factor k , and then translate by $\langle a, b \rangle$; $(x, y) \rightarrow (kx - ka + a, ky - kb + b)$. **14.** Assume the preimage is a triangle, and let the angle measures in the preimage be x , y , and z . By the Triangle Angle Sum Theorem, $x + y + z = 180$. If the angle measures in the image are twice the measures in the preimage, then they are $2x$, $2y$, and $2z$. The image is also a triangle, so $2x + 2y + 2z = 180$. Dividing both sides by 2 gives $x + y + z = 90$. But this contradicts what is known.

16.



18. $\frac{2}{3}$ **20.** $X'(1, 1)$, $Y'(3, 3)$, $Z'(5, -1)$
22. $x = 6$, $y = 15$ **24. a.** No; the lengths are not proportional; $\frac{8}{2} \neq \frac{10}{3}$. **b.** He can enlarge it to 8 in. by 12 in., then crop 2 in. from the length. **26.** 12 ft; If k is the scale factor, then $k \cdot 25 = 30$, so $k = 1.2$. Let x be the distance from the light to the puppet. $1.2x = x + 2$, so $0.2x = 2$, or $x = 10$. The light is 10 feet from the puppet, so it is 12 feet from the screen. **28.** A

Lesson 7-2

2. The similarity transformation shown includes a reflection. **4.** If the point is on the line of reflection and is the center of dilation, then the image and preimage points are the same.
6. $G'(9, -15)$, $H'(3, 6)$, $J'(-3, -18)$
8. No; none of the angles in $EFGH$ have the same measure as $\angle D$.
10. Keegan dilated using A as the center of dilation instead of the origin.
12. yes; use rigid motions to map a vertex of one square onto a vertex of the other square with the sides aligned. Then dilate so that the side length is the same. **14. a.** $\triangle ABC \sim \triangle DEF$; The segment connecting any two midpoints in a triangle is parallel to and half the length of the opposite side. Because of properties of parallel lines and transversals, the angles of the two triangles can be shown to be congruent. The side lengths of $\triangle DEF$ are half the lengths of $\triangle ABC$, so the scale factor is 0.5. **b.** $\triangle ABC$ is reflected across \overleftrightarrow{AC} , dilated with center of dilation B' and scale factor of 0.5, and translated.



Selected Answers

Topic 7

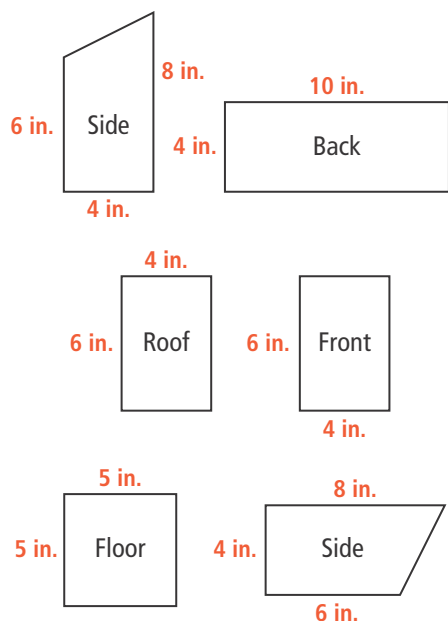
16. $L'(1, -2)$, $M'(2, -2)$, $N'(2, 2)$, $P'(1, 2)$

18. $A'(0.5, -1.5)$, $B'(0, 0)$, $C'(-1.25, -2)$, $D'(-0.5, -2.5)$ **20.** Answers may vary.

Sample: across $x = -3$, dilation with center at A' and scale factor 2; $\triangle ABC \sim \triangle ZYX$ **22.** Yes; it appears you can scale the figure on the left by $\frac{1}{2}$ and map the image onto the figure on the right.

24. He can use the 3 ft by 4 ft paper if he rotates it, because the side lengths are proportional to his cutout. The 180 cm by 120 cm paper does not have side lengths that are proportional to his cutout. He can use it if he cuts it down to 160 cm by 120 cm.

26. Dollhouse Pattern



28. D

Lesson 7-3

2. Allie is incorrect, because the triangles do not meet the conditions of the SAS~ Theorem. The corresponding sides are proportional, but they do not include the congruent angles.

4. $\triangle BCD \sim \triangle XYZ$ by SSS~, and

$\triangle ADB \sim \triangle WZX$ by ASA~. Since

$\triangle ADB \sim \triangle WZX$, $\frac{XW}{WZ} = \frac{BA}{AD}$. Thus,

all corresponding sides of the quadrilaterals are in proportion. By the Angle-Sum Postulate, all of the corresponding vertex angles of the quadrilaterals are congruent, so $ABCD \sim WXYZ$. **6.** Let $AC = BC = x$, and let $PR = QR = y$. Two pairs of corresponding sides are proportional, meaning $\frac{AC}{PR} = \frac{BC}{QR} = \frac{x}{y}$, and the included angles are congruent, meaning $\angle C \cong \angle R$.

Therefore, $\triangle ABC \sim \triangle PQR$ by SAS~.

8. $b = 12$ **10.** Dilate $\triangle XYZ$ by scale factor $\frac{TU}{XZ}$. The image $\triangle X'Y'Z'$ is

congruent to $\triangle TUV$ by SAS, so there is a rigid motion that maps $\triangle X'Y'Z'$ to $\triangle TUV$. Since the composition of a dilation and rigid motion maps $\triangle XYZ$ to $\triangle TUV$, $\triangle XYZ \sim \triangle TUV$. **12.** If the triangles are congruent by ASA, then they have two pairs of congruent angles. That meets the conditions for the AA~ Similarity Theorem, so the triangles are similar. **14.** Dilate $\triangle LMN$

by a factor of $\frac{QR}{LM}$. The image $\triangle L'M'N'$ is congruent to $\triangle QRS$ by ASA, so there is a rigid motion that maps $\triangle L'M'N'$ to $\triangle QRS$. Since the composition of a dilation and a rigid motion maps $\triangle LMN$ to $\triangle QRS$, $\triangle LMN \sim \triangle QRS$.

16. No; $\frac{AB}{DE} = \frac{BC}{EF} = \frac{2}{3}$, but $\frac{AC}{DF} \neq \frac{2}{3}$, so $\triangle ABC$ and $\triangle DEF$ are not similar.

18. Yes; $\angle Y = 180^\circ - 50^\circ - 22^\circ = 108^\circ$ and $\frac{ST}{XY} = \frac{TU}{YZ} = \frac{3}{4}$, so $\triangle STU \sim \triangle XYZ$ by SAS~.

20. 33.8 **22.** 96° , 42° , and 42° ; Answers may vary. Sample: Consider the lower left pane. The lower left angle of this pane is congruent to the



Selected Answers

Topic 7

lower left angle of the entire window. Each pane is identical, so the horizontal edge is $\frac{1}{3}(60) = 20$ ft and the left side is $\frac{1}{3}(40.5) = 13.5$ ft. This equivalent ratio of the sides $\frac{20}{60} = \frac{13.5}{40.5}$ and the congruent angles prove that the panes and the overall window are similar by SAS~. **24.** $1,300 \mu\text{m}$; $x^2 = 300^2 + 720^2$, so $x = 780 \mu\text{m}$. The remaining cable distance is $1,200 - 780 = 480 \mu\text{m}$.

$$\frac{y}{780} = \frac{480}{720}, \text{ so } y = 520 \mu\text{m}. x + y =$$

$$780 + 520 = 1,300 \mu\text{m}. \quad \mathbf{26. C}$$

Lesson 7-4

2. Leg \overline{JK} is the short leg of $\triangle JKL$. Length JK is the geometric mean of the length of the hypotenuse, KL , and the length of the segment of the hypotenuse that is adjacent to \overline{JK} , which is KM . The proportion should be $\frac{KL}{JK} = \frac{JK}{KM}$. **4.** 3 **6.** 30 **8.** $2\sqrt{5}$ **10.** 6 in.;

The bracket is a right triangle with an altitude to the hypotenuse. The shelf cannot be wider than the short leg of the bracket. The length of the short leg is the geometric mean of the length of the hypotenuse and the 4-in. segment.

$$\frac{4}{x} = \frac{x}{(4+5)}$$

$$x^2 = 36$$

$$x = 6$$

The maximum shelf width is 6 in.

12. Amaya used the incorrect correspondence of side lengths.

14. a. $\angle JML$ is a right angle because $\overline{LM} \perp \overline{JK}$, so $\angle JML$ is congruent to $\angle JLK$. $\angle L$ is in both $\triangle JKL$ and $\triangle JLM$ (and is congruent to itself). Thus, $\triangle JLM \sim \triangle JKL$ by AA~. $\angle LMK$ is a right angle because $\overline{LM} \perp \overline{JK}$, so $\angle LMK$ is congruent to $\angle JLK$.

$\angle K$ is in both $\triangle JKL$ and $\triangle LKM$ (and is congruent to itself). Thus, $\triangle LKM \sim \triangle JKL$ by AA~. **b.** Corresponding parts of similar triangles are proportional by the definition of similarity. \overline{JM} in $\triangle JLM$ corresponds to \overline{LM} in $\triangle LKM$ (both shorter legs), and \overline{LM} in $\triangle JLM$ corresponds to \overline{KM} in $\triangle LKM$ (both longer legs). Thus, $\frac{JM}{LM} = \frac{LM}{KM}$.

c. Corresponding parts of similar triangles are proportional by the definition of similarity. \overline{JK} in $\triangle JKL$ corresponds to \overline{JL} in $\triangle JLM$ (both hypotenuses), and \overline{JL} in $\triangle JKL$

corresponds to \overline{JM} in $\triangle JLM$ (both shorter legs). Thus, $\frac{JK}{JL} = \frac{JL}{JM}$. \overline{JK} in $\triangle JKL$ corresponds to \overline{LK} in $\triangle LKM$ (both hypotenuses), and \overline{LK} in $\triangle JKL$ corresponds to \overline{MK} in $\triangle LKM$ (both longer legs). Thus, $\frac{JK}{LK} = \frac{LK}{MK}$.

16. $\triangle ADB$ and $\triangle BDC$; $\angle ADB \cong \angle ABC$ and $\angle DAB \cong \angle BAC$ (or $\angle A \cong \angle A$), so $\triangle ADB \sim \triangle ABC$ by AA~. $\angle BDC \cong \angle ABC$ and $\angle BCD \cong \angle ACB$ (or $\angle C \cong \angle C$), so $\triangle BDC \sim \triangle ABC$ by AA~. **18.** $6\sqrt{2}$; By Corollary 1 to Theorem 7-4, $\frac{6}{y} = \frac{y}{12}$, so $y^2 = (6)(12) = 72$ and $y = \sqrt{72} = 6\sqrt{2}$ (because distance must be positive).

20. a. 8; By Corollary 1 to Theorem 7-4, $\frac{m-4}{8} = \frac{8}{2m}$, so $(m-4)(2m) = (8)(8)$, and $2m^2 - 8m - 64 = 0$. Thus, $m = 8$. The solution $m = -4$ is extraneous, because distance must be positive. **b.** 12; By Corollary 2 to Theorem 7-4, $\frac{9}{n+3} = \frac{n+3}{9+7}$, so $(n+3)^2 = (9)(16)$, and $n^2 + 6n - 135 = 0$. Thus, $n = 9$. The solution $n = -15$ is extraneous because distance must be positive.



Selected Answers

Topic 7

22. 96 ft.; $\frac{h}{160} = \frac{120}{200}$, so $h = 96$.

24. The altitude \overline{XZ} to the hypotenuse of a right triangle \overline{WY} forms a right angle with the hypotenuse. By definition of perpendicular lines, $\overline{XZ} \perp \overline{WY}$ since these segments intersect and form right angles. Let the slope of $\overline{XZ} = m$. By definition, the slope of the perpendicular line \overline{WY} is the negative reciprocal, $\frac{-1}{m}$. $m(\frac{-1}{m}) = -1$. Therefore, the product of the slopes of two perpendicular lines is equal to -1 .

26. D

Lesson 7-5

2. Carmen is incorrect, because the segments are proportional to the adjacent sides, and the adjacent sides are not congruent, so $AD \neq BD$.

4. Isosceles triangle; By the Triangle-Angle-Bisector Theorem, $\frac{PR}{RS} = \frac{PQ}{QS}$.

Since $\overline{RS} \cong \overline{QS}$, $RS = QS$, so $PR = PQ$, and the triangle is isosceles by definition.

6. 15 **8.** 9.75 **10.** 16 **12.** Benson should write the proportion $\frac{10}{x} = \frac{7}{5}$, so the solution is $x \approx 7.14$. **14.** By the Corresponding Angles Theorem, $\angle BMN \cong \angle BAC$, and $\angle BNM \cong \angle BCA$. By AA~, $\triangle BMN \cong \triangle BAC$. Because the triangles are similar, $\frac{BA}{BM} = \frac{BC}{BN}$, or $\frac{BM + MA}{BM} = \frac{BN + NC}{BN}$, or $1 + \frac{MA}{BM} = 1 + \frac{NC}{BN}$, $\frac{AM}{MB} = \frac{CN}{NB}$.

16. By the Side-Splitter Theorem, $\frac{CA}{AE} = \frac{CD}{DB}$. By the Corresponding Angles

Theorem, $\angle AEB \cong \angle CAD$. By the definition of bisector, $\angle CAD \cong \angle DAB$. Since they are alternate interior angles, $\angle DAB \cong \angle ABE$. By the Transitive

Property of Congruence, $\angle AEB \cong \angle ABE$, so $\triangle AEB$ is isosceles, and $AE = AB$.

Substitute AB for AE in the proportion to get $\frac{CD}{DB} = \frac{CA}{AB}$. **18.** 3 **20.** 8 **22.** 9.3

24. $\frac{EG}{ED} = \frac{EG}{EG + GD} = \frac{EG}{2EG} = \frac{1}{2}$, $\frac{EH}{HF} = \frac{EH}{EH + HF} = \frac{EH}{2EH} = \frac{1}{2}$, and $\angle E \cong \angle E$, so $\triangle EGN \cong$

$\triangle EDF$ by SAS~. Since corresponding angles of similar triangles are congruent, $\angle EGN \cong \angle EDF$, so $\overline{GH} \cong \overline{DF}$ by the Converse of the Corresponding Angles Theorem. Since corresponding sides of similar triangles are proportional, $\frac{GH}{DF} = \frac{EG}{ED} = \frac{1}{2}$, so $GH = \frac{1}{2}DF$.

26. $KO = \frac{2}{3}JP$, $LM = \frac{1}{3}JP$ **28.** Aisha can use the Triangle-Angle-Bisector Theorem; $6.4 \text{ ft} < AG = 8$

so $\frac{10}{8} = \frac{8}{x} \rightarrow 10x = 64 \rightarrow x = 6.4 >$

30 C

Topic Review

2. similar **4.** center of dilation

6. $F'(2.5, -1)$, $G'(-1, -2)$, $H'(0, 3)$

8. $R_m \circ D_{\frac{1}{3}}$, where m is the line with equation $y = -\frac{1}{2}$ **10.** $\frac{FG}{JG} = \frac{9}{6} = \frac{3}{2}$ and

$\frac{GJ}{GH} = \frac{6}{4} = \frac{3}{2}$, so $\frac{FG}{JG} = \frac{GJ}{GH}$. $m\angle FGJ = m\angle JGH = 90^\circ$, so $\angle FGJ \cong \angle JGH$. Thus,

$\triangle FGJ \sim \triangle JGH$ by SAS~. **12.** Since $m\angle U = 180^\circ - m\angle T - m\angle V = 78^\circ$,

$\angle U \cong \angle Z$. So triangle similarity can be shown by AA~ if $m\angle X = m\angle T = 37^\circ$ or if $m\angle Y = m\angle V = 65^\circ$. **14.** $8\sqrt{3}$

16. 48 **18.** 4 **20.** GK and GH ; By the similarity $\triangle GHJ \cong \triangle GJK$, $\frac{GH}{GJ} = \frac{GJ}{GK}$.



Selected Answers

Topic 8

20. The hypotenuse of a right triangle, c , is always longer than the length of the legs, a and b . Thus, the sine and cosine ratios must be less than one. **22.** $\frac{b}{c}$ **24.** $\frac{\sqrt{17}}{17}$ **26.** $\frac{1}{4}$ **28.** $\frac{\sqrt{17}}{17}$ **30.** $\frac{1}{2}$
32. $\frac{\sqrt{2}}{2}$ **34.** $\frac{\sqrt{3}}{2}$ **36.** ≈ 5.41 **38.** $\approx 36.87^\circ$
40. $\approx 18.43^\circ$ **42.** $\approx 33.75^\circ$ **44.** Using the sine ratio, the boom is approximately 70 ft long. The distance from the bottom of the boom to the top of the building is approximately 70.5 ft, so the boom itself does not reach the edge of the building, but if you include the length of the platform, it should.
46. 97 ft **48.** D

Lesson 8-3

2. Amelia used sides adjacent to the angles, rather than sides opposite the angles. **4.** No; to use the Law of Sines you must know at least one side length opposite a known angle measure.
6. $m\angle J$, GJ **8.** 46.5 **10.** 10.9 **12.** 23.5
14. No; because you do not know the side opposite either of the known angles. **16.** Using the Distance Formula, $\sqrt{18} = 4.2426\dots$ Using the Law of Sines, the value was 4.2404.... The numbers match to the hundredths. The angle measures given in the diagram may be rounded, which may account for the difference. **18.** 5.8 **20.** 4.3 **22.** 12.4
24. 42.4° **26.** 57.0° **28.** 34.0° **30.** 70.6
32. 26.8 **34.** 23 ft **36.** No; using the Law of Sines, the distance from point Y to the pier is approximately 4.0293 miles. Using the definition of the sine ratio, the distance from the pier to the line of travel is approximately 3.8098. The boat will pass more than a mile outside the restricted area. **38.** C

Lesson 8-4

2. Cameron used $m\angle D$ in place of $m\angle F$. Cameron should have used the Law of Sines. **4.** No, at least one side length must be given. **6.** h **8.** $m\angle K$, $m\angle L$, $m\angle M$
10. 12.7 **12.** 5.0 **14.** Tavon forgot to take the square root to find the final answer, about 6.99. **16.** You use the Law of Sines if you know a side opposite a known angle, otherwise you use the Law of Cosines. **18.** $e = \sqrt{d^2 + f^2 - 2df \cos E}$ **20.** 23.3 **22.** 42.3
24. 59.7 **26.** 19.7 **28.** 69.7 yd **30.** 1 hour, 3 minutes **32.** B, D **34. Part A** No; use the Law of Cosines to find that the distance from Island Port to Port A is about 37.6 mi **Part B** 28.6 mi; yes **Part C** no; 0.9° south of west

Lesson 8-5

2. Jamie used the wrong sides in the calculation. Since the angle between the 14 in. and 10 in. sides is known, those are the side lengths that should be used. **4.** First use the Law of Cosines to find the measure of any angle. Then use the formula $A = \frac{1}{2}bc \sin A$. **6.** 916.9 ft
8. 84 units² **10.** The area is found using the formula $A = \frac{1}{2}bc \sin A$. **12.** 5.1; 2.6 **14.** $A = 2 \cdot \frac{1}{2}ab \sin(180 - x)^\circ$ $A = ab \sin(180 - x)^\circ$ **16.** 30° **18.** 157.7 ft
20. 16.3 units² **22.** 12.0 units² **24.** No; $\tan(0.055) \cdot 7,000 \text{ cm} \approx 6.72 \text{ cm}$ which is greater than 6.1 cm, the radius of the target, so Benito's arrow will strike below the yellow circle. **26.** 8.54
28. Part A 20.8° **Part B** 20.9 m **Part C** 19.3°



Selected Answers

Topic 8

Topic Review

2. cosine **4.** angle of elevation **6.** $\sqrt{39}$
8. 51.3° **10.** If you know XY , you could use the Pythagorean Theorem to find YZ . Or, if you know $m\angle X$ or $m\angle Y$, you could use the tangent of the angle measure to find YZ . **12.** 6.4 **14.** 17.3°

16. The perimeter is $BC + AC + AB$. By the Law of Sines, $\frac{\sin C}{AB} = \frac{\sin A}{BC}$, so $BC = \frac{AB \sin A}{\sin C}$. Similarly, $\frac{\sin B}{AC} = \frac{\sin A}{BC}$, so $AC = \frac{BC \sin B}{\sin A}$. Also, $\frac{\sin C}{AB} = \frac{\sin A}{BC}$, so $AB = \frac{BC \sin C}{\sin A}$. So, the perimeter can be written as $\frac{AB \sin A}{\sin C} + \frac{BC \sin B}{\sin A} + \frac{BC \sin C}{\sin A}$. **18.** 35.8 cm^2
20. 74.7 in.^2



Selected Answers

Topic 9

Lesson 9-1

2. Chen could also use the Pythagorean Theorem. Since AB is 4 and AC is 3,

$$4^2 + 3^2 = (BC)^2$$

$$25 = (BC)^2$$

$$5 = BC$$

4. $2(\sqrt{10} + \sqrt{17})$ **6.** Parallelogram; because the diagonals bisect each other, $JKLM$ is a parallelogram.

8. 12.5 square units **10.** $y = -\frac{5}{4}x + \frac{1}{2}$

12. Use the Slope Formula to show the diagonals are perpendicular to each other. Then find the midpoints of each diagonal and the equation of the line containing each diagonal. Show the midpoint of one diagonal is a point on the other diagonal. Then one diagonal is the perpendicular bisector of the other. By the Perpendicular Bisector Theorem, each endpoint of the diagonal that is the perpendicular bisector is equidistant from the endpoints of the other diagonal, so two pairs of consecutive sides are congruent. Then the quadrilateral is a kite by definition. **14.** $\ell \parallel n$

16. $\sqrt{45}$ units **18.** Isosceles, right triangle; it is isosceles because $AB = BC = \sqrt{10}$ units and $AC = 2\sqrt{5}$ units. It is a right triangle because the slope of the line through $\overline{AB} = \frac{1}{3}$, and the slope of the line through $\overline{BC} = -3$, so $\overline{AB} \perp \overline{BC}$ and $\angle ABC = 90^\circ$.

20. Parallelogram; $AC = DE = 2\sqrt{5}$ and $AD = CE = 5$, so $ACED$ is not a rhombus; $AE = \sqrt{65}$ and $CD = 5$, so $ACED$ is not a rectangle. **22.** area = 5 square units; perimeter = $2\sqrt{10} + 2\sqrt{5}$ units

24. $(-3, -1)$; rhombus **26.** $(\frac{98}{17}, \frac{120}{17})$

28. Neither is correct. Each explored a triangle with height 3 units and base 8 units so the areas are the same. **30.** A

Lesson 9-2

2. Venetta used horizontal distance instead of vertical distance and vice versa. **4.** Planning a proof before starting makes it easier to complete the proof, since you have already decided how the proof will be done and just need to do the calculations. **6.** Draw rhombus $ABCD$ on a coordinate plane with vertices $A(0, 0)$, $B(b, \sqrt{a^2 - b^2})$, $C(a + b, \sqrt{a^2 - b^2})$, and $D(a, 0)$. Find the slopes of the two diagonals and show that the product of the slopes is -1 .

8. Draw the triangle on the coordinate plane. Write equations for the lines containing the altitudes and show that they intersect at a common point.

10. Draw $ABCD$ with $A(-a, 0)$, $B(0, b)$, $C(a, 0)$, and $D(0, -c)$. By the Distance Formula, $AB = BC = \sqrt{a^2 + b^2}$ and $AD = CD = \sqrt{a^2 + c^2}$. $ABCD$ is a kite because it is a quadrilateral with two pairs of consecutive congruent sides, and values for a , b , and c can be chosen for any lengths. The diagonals of the kite are the coordinate axes, which are perpendicular. **12.** By the Distance Formula, $AC = BD = \sqrt{(a + b)^2 + c^2}$. The diagonals have equal length and are thus congruent. **14.** Tonya should use (b, b) to represent any point on the perpendicular bisector since (a, a) is one specific point. **16.** Find the equation of the line containing \overline{BD} . Show that the coordinates of point E are a solution of



Selected Answers

Topic 9

the equation. **18.** Draw a parallelogram $ABCD$ on a coordinate plane with vertex A at the origin, vertex $B(a, 0)$, vertex $C(b + a, c)$, and vertex $D(b, c)$; specify that $AD \neq AB$. Use the distance formula to find the lengths of the diagonals and show they are equal only if $AD = AB$. **20.** Draw a parallelogram $ABCD$ on a coordinate plane with vertex A at the origin, vertex $B(a, 0)$, vertex $C(b + a, c)$, and vertex $D(b, c)$; specify that $AD \neq AB$. Use the midpoint formula to find the midpoints of the diagonals and show that they are the same point. **22.** Draw a rectangle $ABCD$ on a coordinate plane with vertex A at the origin, vertex $B(b, 0)$, vertex $C(b, a)$, and vertex $D(0, a)$. Using the Distance Formula, $AB = b$, $BC = a$, and $AC = \sqrt{a^2 + b^2}$. Substituting, $AC = \sqrt{AB^2 + BC^2}$, so square both sides to get $AC^2 = AB^2 + BC^2$. **24.** Draw kite $ABCD$ on a coordinate plane with vertex $A(-a, 0)$, vertex $B(0, b)$, vertex $C(a, 0)$, vertex and vertex $D(0, -c)$; specify that $b \neq c$. This represents any kite with the side lengths $\sqrt{a^2 + b^2}$ and $\sqrt{a^2 + c^2}$ by choice of a , b , and c . Using the Midpoint Formula, the midpoint of \overline{AC} is $(0, 0)$ and the midpoint of \overline{BD} is $(0, \frac{b-c}{2})$. Since $b \neq c$, the midpoint of \overline{BD} is not a point on \overline{AC} , but the midpoint of \overline{AC} is a point on \overline{BD} . So only diagonal \overline{BD} bisects diagonal \overline{AC} . **26.** $(4, \frac{8}{3})$
28. \$109.97 **30.** 4 **32.** C

Lesson 9-3

2. The equation should contain the value of the radius squared; Leo did

not square the radius. **4.** Find the midpoint of the diameter; this is the center of the circle. Compute the distance between the center and either endpoint to find the radius. Then substitute the radius and the coordinates of the center in the equation of a circle.

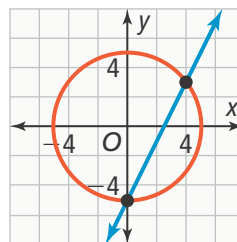
6. $(x - 6)^2 + (y - 2)^2 = 64$

8. $(x + 9)^2 + (y - 5)^2 = 16$

10. $(x + 1)^2 + (y - 2)^2 = 8$

12. $(x - 5)^2 + (y - 11)^2 = 185$

14. $(4, 3)$ and $(0, -5)$;



16. The graph is the point (a, b) .

18. center: $(\frac{a+c}{2}, \frac{b+d}{2})$,

radius: $\sqrt{\frac{(a-c)^2 + (b-d)^2}{2}}$

20. center: $(4, -3)$, radius: 8

22. center: $(-5, -11)$, radius: $4\sqrt{2}$

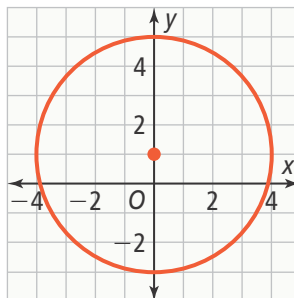
24. $(x + 1)^2 + (y + 2)^2 = 5$

26. $(x - 5)^2 + (y - 1)^2 = 16$

28. $(x - 2)^2 + (y + 4)^2 = 125$

30. no **32.** no

34.





Selected Answers

Topic 9

36. 3, -5 **38.** 7:00 AM; The wave travels at 7 mi/min. The distance between the epicenter and Port Charles is about 500 mi. $500 \div 7 \approx 72$, so the wave will reach Port Charles in about 72 min.

The distance between Port Charles and the Epicenter is 500 miles.

$$T = \frac{D}{R} = \frac{500}{7} = 71.4286$$

So the wave will reach the Port Charles after roughly 72 minutes.

$$5:48 + 1:12 = 7:00 \text{ AM}$$

40. No; the semicircle has radius 27 ft and center in the middle of the road at (0, 0). The equation is $x^2 + y^2 = 27^2$. The x-coordinate of the edge of the outer lane is 24; so solve $24^2 + y^2 = 27^2$ to get $y \approx 12.4$. The height of the tunnel is 12.4 ft at the outer lane. **42.** D

Lesson 9-4

2. Arthur used the distance between the focus and directrix in his equation instead of half this distance. **4.** $(2a, a)$; The equation of the parabola is $y = \frac{1}{4a}x^2$. The y-coordinate of P is a,

so solve to find the x-coordinate:

$$a = \frac{1}{4a}x^2, \text{ so } 4a^2 = x^2, \text{ or } x = 2a.$$

$$\mathbf{6.} y = \frac{1}{16}x^2 \quad \mathbf{8.} y + 2 = \frac{1}{8}(x - 4)^2$$

10. vertex: (0, 0), focus: (0, 2), directrix: $y = -2$ **12.** vertex: (1, 6), focus: (1, 7), directrix: $y = 5$ **14.** $y + 4 = \frac{1}{24}(x - 6)^2$

16. $y + 2 = \frac{1}{8}(x - 4)^2$ **18. a.** focus: (0, 9), directrix: $y = -9$ **b.** distance from (12, 4) to (0, 9): $\sqrt{12^2 + (4 - 9)^2} = 13$; distance from (12, 4) to $y = -9$: $4 - (-9) = 13$ **20.** Sample: A parabola is the set of points such that each point is equidistant from a point called the

focus and a line called the directrix.

22. As the value of p increases, the parabola gets flatter. **24.** (3, 3)

26. $(-4, \frac{3}{2})$ **28.** vertex: (3, 7), focus:

(3, 9), directrix: $y = 5$ **30.** vertex:

(-6, 3), focus: (-6, 7), directrix: $y = -1$

$$\mathbf{32.} y - 2 = \frac{1}{8}x^2 \quad \mathbf{34.} y - \frac{7}{2} = \frac{1}{6}(x + 4)^2$$

$$\mathbf{36.} y - 5 = \frac{1}{16}(x + 4)^2 \quad \mathbf{38.} (1, 2)$$

$$\mathbf{40.} (1, 3) \quad \mathbf{42.} y = \frac{1}{16}x^2 \quad \mathbf{44.} \text{B, C, D}$$

46. Part A Place the parabola so that the vertex is at the origin and the upper corners are at (-2, 3) and (2, 3).

$$y = \frac{3}{4}x^2 \quad \mathbf{Part B} \frac{1}{3} \text{ in. above the}$$

vertex **Part C** wider; The greater the value of p , the lesser the value of $\frac{1}{4p}$.

Since the equation is $y = \frac{1}{4p}x^2$, a greater value of p will produce a lesser value of y for the same x -value. So, to get the same y -value, the values of x will need to be greater, which means the mirror is wider.

Topic Review

2. focus **4.** directrix **6.** no **8.** area: 20, perimeter: $6\sqrt{10}$ **10.** $C(t + p, q)$

12. $X(-j, -k)$ **14.** Yes; since $PX = PY$,

$$\sqrt{c^2 + d^2} = \sqrt{(c - a)^2 + d^2}, \text{ so } c^2 =$$

$$(c - a)^2, \text{ or } c^2 = c^2 - 2ac + a^2,$$

and $2ac = a^2$, which means that $c = \frac{a}{2}$ and the coordinates of P are $(\frac{a}{2}, d)$.

The midpoint M of \overline{XY} is $(\frac{a}{2}, 0)$.

Since P and M have the same x-coordinate, \overline{PM} is vertical. Since X and Y have the same y-coordinate, \overline{XY} is horizontal. Therefore, the new fence is perpendicular to \overline{XY} .



Selected Answers

Topic 9

16. $(x + 2)^2 + (y - 3)^2 = 25$ **18.** yes

20. $(x - 7)^2 + (y - 7)^2 = 100$

22. $y + 3 = \frac{1}{12}x^2$ **24.** $4p$; Points A and B have y -coordinates of p , so the distance of both points to the directrix is $2p$. Therefore, both points are $2p$ from F , so $AB = 4p$.



Selected Answers

Topic 10

Lesson 10-1

2. Luke used the formula for arc length for central angles or arc measures in degrees, but the diagram gives the central angle in radians. **4.** A quarter of a circle corresponds to a central angle of 90° . A sector with a central angle of 90° has an area of $\frac{90}{360}\pi r^2 = \frac{1}{4}\pi r^2$. **6.** measure: 277° ; length: $\frac{277}{45}\pi$ **8.** $\frac{320}{9}\pi$ **10.** No; arc length depends on arc measure and radius of the circle, so if the circles have different radii, the arc measures will be different. **12.** 90° ; Circle T has center $(2, 3)$ and radius 5. \overline{XT} has slope $-\frac{3}{4}$ and \overline{YT} has slope $\frac{4}{3}$, so these radii are perpendicular. **14.** The length of the arc is given by the equation $s = \frac{a}{360}2\pi r$. For any value of a , the expression $\frac{a}{360}2\pi r$ is a constant, so this represents a proportional relationship. **16.** 100° **18.** 125° **20.** $\frac{44}{9}\pi$ **22.** 65.3 **24.** 74.3 **26.** 20.6 ft **28.** 20.2 cm **30.** 16 **32. Part A** 765 ft^2 ; The stage is a rectangle and a circle, but one segment of the circle is included in both of these parts so it needed to be subtracted to find the area. The central angle of the segment is about 73.7° , and the area of the triangle corresponding to the segment is 75 ft^2 . **Part B** 102.5 ft; From Part A, the measure of the arc not included in the stage is 73.7° . The length of the string of lights is: $20 + \frac{360 - 73.7}{360}2\pi(12.5) + 20 \approx 102.5$. **Part C** 130; The area of the sector would be about 177 ft^2 , and the length of the segment corresponding to the sector would be about 22.6 ft.

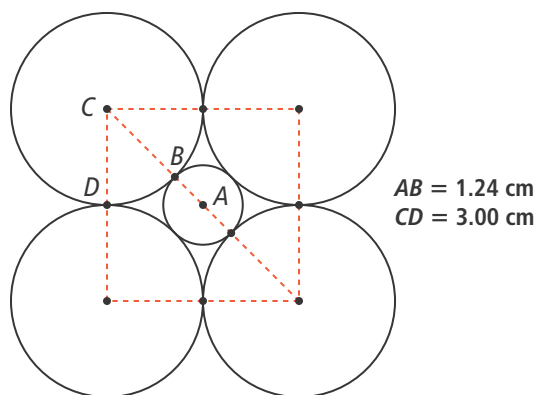
Lesson 10-2

2. It appears that A is the only point of intersection, but by the Triangle Angle Sum Theorem, $m\angle A = 89^\circ$, so \overline{AB} is not perpendicular to \overline{AG} , so \overline{AB} is not a tangent. **4.** Since $\angle APQ$, $\angle QSA$, $\angle QSB$, and $\angle QRB$ are right angles, $\angle PAS$ and $\angle PQS$ are supplementary, and $\angle RBS$ and $\angle RQS$ are supplementary. But since $\angle PQS$ and $\angle RQS$ form a linear pair, they are also supplementary. So, $\angle PAS \cong \angle RQS$ and $\angle RBS \cong \angle PQS$. **6.** yes **8.** $2\sqrt{14}$ **10.** 72° **12. Given:** Line m is tangent to $\odot T$ at G . **Prove:** $\overline{GT} \perp m$ **Proof:** Assume \overline{GT} is not perpendicular to m . Draw a perpendicular segment from T to line m , and let H be the point of intersection. Then $\triangle HGT$ is a right triangle. Since \overline{GT} is the side opposite the right angle, it is the hypotenuse. Therefore, $GT > HT$. But since G is the only point of intersection of $\odot T$ and m , H is on the exterior of $\odot T$. Since \overline{GT} is a radius, $GT < HT$. But this contradicts $GT > HT$. Therefore, the assumption is false and $\overline{GT} \perp m$. **14. Given:** $\odot T$ with tangent \overline{DE} and point of tangency E , and tangent \overline{DF} and point of tangency F **Prove:** $\overline{DE} \cong \overline{DF}$ **Proof:** Since \overline{TE} and \overline{TF} are both radii of $\odot T$, $\overline{TE} \cong \overline{TF}$. Since \overline{DE} is a tangent at E , $\angle TED$ is a right angle. Similarly, $\angle TFD$ is a right angle. Draw \overline{TD} . $\overline{TD} \cong \overline{TD}$, and \overline{TD} is the hypotenuse of right triangles $\triangle TED$ and $\triangle TFD$, so by HL, $\triangle TED \cong \triangle TFD$. Therefore, by CPCTC, $\overline{DE} \cong \overline{DF}$. **16.** $6\sqrt{3}$ **18.** 9 **20.** 69 **22.** 60 **24.** 72.6 km **26.** D, E **28. Part A** $(\sqrt{2} - 1)r$ **Part B** Answers may vary. Sample:



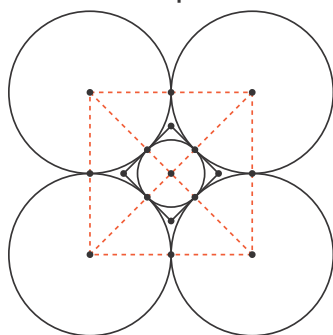
Selected Answers

Topic 10



Yes; if the radius of the larger circle is 3, then the radius of the smaller circle is 1.24. $(\sqrt{2} - 1)3 \approx 1.24$

Part C Sample:



The tangent lines are perpendicular to the radii of the circles at the point of tangency. When two tangents to the same circle intersect, the distances from the point of intersection to the points of tangency are equal.

Lesson 10-3

2. Sasha did not make sure the angles were central angles. **4.** The radius of the circle is 5.5, so $QT = 5.5$. You can use the Pythagorean Theorem to find half of UT and then double that value to get UT . **6.** 86° **8.** 43° **10.** 86°

12. It is given that $\overline{AB} \cong \overline{CD}$. Because all radii of a circle are congruent, $\overline{EA} \cong \overline{EB} \cong \overline{EC} \cong \overline{ED}$. So by SSS, $\triangle AEB \cong \triangle CED$. Therefore, by CPCTC,

$\angle AEB \cong \angle CED$. **14.** Ashton did not apply the Pythagorean Theorem correctly. The correct relationship is $(PS)^2 + (TS)^2 = (PT)^2$. He also did not use the correct value for PS . **16.** It is given that \overline{LN} is a diameter of $\odot Q$ and $\overline{KP} \cong \overline{MP}$. Since all radii of a circle are congruent, $\overline{QM} \cong \overline{QK}$, so by the Converse of the Perpendicular Bisector Theorem, Q lies on the perpendicular bisector of \overline{KM} . Since P is the midpoint of \overline{KM} , P lies on the perpendicular bisector of \overline{KM} . Points P and Q both lie on \overline{LN} , so \overline{LN} is the perpendicular bisector of \overline{KM} . Therefore, by the definition of perpendicular bisector, $\overline{LN} \perp \overline{KM}$. **18.** 90° **20.** $\angle TBU$ **22.** Check students' work. When constructing the square, you use the diameter of the circle and its perpendicular bisector, not the radius like when constructing the hexagon and equilateral triangle. Also, when drawing the square, you connect 4 vertices on the circle, whereas you connect 6 vertices when drawing a hexagon and 3 vertices when drawing a triangle. **24.** 42 **26.** ≈ 19.09 in. **28.** It is given that \overline{QR} is a chord of $\odot P$ and \overline{AB} is the perpendicular bisector of \overline{QR} . Since all radii of a circle are congruent, $\overline{PQ} \cong \overline{PR}$, and by the Converse of the Perpendicular Bisector Theorem, P lies on the perpendicular bisector of \overline{QR} . Therefore, \overline{AB} contains P . **30.** Yes; the diameter of a wheel that fits exactly in the largest pothole is 25 in. **32.** A, B, D **34. Part A** 64 mi **Part B** ≈ 38.42 mi **Part C** 1:18:40 P.M. A chord that is 40 miles from the center has a length of 60 miles. The plane travels 60 miles in 6 minutes 40 seconds.



Selected Answers

Topic 10

Lesson 10-4

2. Darren's answer is incorrect because he says the measure of an intercepted arc is one-half the measure of the inscribed angle. He should have said that the measure of the inscribed angle is one-half the measure of the intercepted arc. **4.** Yes; since $RSTU$ is a quadrilateral inscribed in a circle, $\angle RST$ and $\angle TUR$ are supplementary. So, $m\angle TUR = 109^\circ$. The arc intercepted by $\angle TUR$ is \widehat{RST} . By the Inscribed Angles Theorem, $m\widehat{RST} = 218^\circ$. **6.** 142°
8. 97° **10.** 110° **12.** 55° **14.** 74° **16.** 47°
18. $m\widehat{WXZ} = 2(m\angle WZY) = 142^\circ$, so $m\widehat{WVZ} = 360^\circ - 142^\circ = 218^\circ$. **20.** No; \widehat{GJK} is less than a semicircle, so its measure is less than 180° . Because $\angle HGK$ intercepts \widehat{GJK} , $m\angle HGK$ is less than one-half of 180° , or less than 90° . **22.** 45° **24.** 100.5° **26.** 134°
28. 30° **30.** 48° **32.** Draw \overline{CR} . Because all radii in a circle are congruent, $\overline{CR} \cong \overline{CS}$, so $\triangle CRS$ is isosceles by definition. Applying the Isosceles Triangle Theorem, $\angle CRS \cong \angle CSR$. By the Triangle Exterior Angle Theorem, $m\angle TCR = m\angle CRS + m\angle CSR$. By substitution, $m\angle TCR = 2(m\angle CSR)$, so $\frac{1}{2}m\angle TCR = m\angle CSR = m\angle RST$. Thus, $m\angle RST = \frac{1}{2}m\widehat{RT}$ because the measure of an arc is equal to the measure of the corresponding central angle. **34.** Draw \overline{BP} and label the point of intersection E with $\odot P$. By Theorem 10-1, $m\angle ABE = 90^\circ$. Using the Angle Addition Postulate, $m\angle ABD + m\angle DBE = m\angle ABE$, so $m\angle ABD + m\angle DBE = 90^\circ$, or $m\angle DBE = 90^\circ - m\angle ABD$. From the Inscribed Angles Theorem, $m\angle DBE = \frac{1}{2}m\widehat{DE}$, so $90^\circ - m\angle ABD = \frac{1}{2}m\widehat{DE}$ from the Transitive Property of Equality. By

the addition of arc angle measures, $m\widehat{BCD} + m\widehat{DE} = 180^\circ$, so $m\widehat{DE} = 180^\circ - m\widehat{BCD}$. Using substitution and simplifying, $90^\circ - m\angle ABD = 90^\circ - \frac{1}{2}m\widehat{BCD}$ and $m\angle ABD = \frac{1}{2}m\widehat{BCD}$. **36.** Yes; the measure of the inscribed angle is one-half the measure of the intercepted arc. The curved edge of the plate is a 72° arc with length 6 in. $6 = \frac{72}{360} \cdot 2\pi r$, so the radius is about 4.77 in. and the diameter is about 9.54 in. **38.** $(180 - \frac{1}{2}x)$ **40. Part A** Yes; $\triangle AED$, $\triangle BEF$, and $\triangle CFD$ are isosceles because the distances to two points of tangency on the same circle from an exterior point are equal. **Part B** No; using variables for each base angle of the isosceles triangles and the straight angles formed at the points of tangency, set up and solve a system of three equations with three variables. From this, $m\angle A = 40^\circ$, $m\angle B = 80^\circ$, and $m\angle C = 60^\circ$. Since the vertex angles of the triangles are not congruent, the triangles are not similar.

Lesson 10-5

2. The product of the segment lengths of one secant equals the product of the segment lengths of the other secant. The first line of the solution should be $GK \cdot KJ = FK \cdot KH$. **4.** $m\angle 1 = \frac{1}{2}(x - (360 - x))^\circ = \frac{1}{2}(2x - 360)^\circ = (x - 180)^\circ$ **6.** 55° **8.** 12.6 **10.** 6
12. $m\angle VXZ$ is half of the difference of the measures of the intercepted arcs, not half the sum of the measures of the intercepted arcs. **14.** 36° **16.** 93°
18. Given: $\odot A$, secants \overline{PR} and \overline{QS} intersecting at point T **Prove:** $m\angle PTQ$



Selected Answers

Topic 10

$= \frac{1}{2}(m\widehat{RS} + m\widehat{PQ})$ **Proof:** Draw \overline{QR} to form $\triangle QRT$. $\angle QRP$ is an inscribed angle, so $m\angle QRP = \frac{1}{2}m\widehat{PQ}$. $\angle RQS$ is an inscribed angle, so $m\angle RQS = \frac{1}{2}m\widehat{RS}$. By the Triangle Exterior Angle Theorem, $m\angle PTQ = m\angle QRP + m\angle RQS$. Substitute the equivalent arc measures and use the Distributive Property to get $m\angle PTQ = \frac{1}{2}(m\widehat{RS} + m\widehat{PQ})$. **20.** 32° **22.** 13

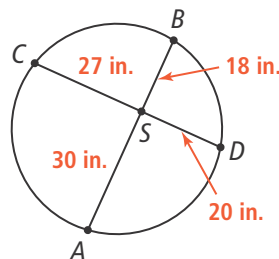
24. Given: $\odot C$, secant \overleftrightarrow{QS} , and tangent \overline{PQ} **Prove:** $(PQ)^2 = (RQ)(SQ)$ **Proof:** Draw \overline{PS} and \overline{PR} . By the Inscribed Angles Theorem, $m\angle PSQ = \frac{1}{2}m\widehat{PR}$.

By Theorem 10-9, $m\angle QPR = \frac{1}{2}m\widehat{PR}$. So using the Transitive Property of Equality, $m\angle PSQ = m\angle QPR$ so $\triangle PSQ \cong \triangle QPR$. $\angle Q \cong \angle Q$ by the Reflexive Property of Congruence. Thus, $\triangle PQR \sim \triangle SQP$ by AA \sim . Since the ratios of corresponding sides of similar triangles are equal, $\frac{PQ}{SQ} = \frac{RQ}{PQ}$. Using the Multiplication Property of Equality, $(PQ)^2 = (RQ)(SQ)$. **26.** about 38.3%; The measure of the angle formed by two tangents to a circle equals one-half the difference between the measures of the intercepted arcs. If x is the measure of the visible arc of the equator, then $42 = \left(\frac{1}{2}\right)[(360 - x) - x]$, so $x = 138$.

$138 \div 360 \approx 0.383$. **28.** 44°

30. Part A Charles, Benson, Deon, Alberto; Find the combination of distances so that the product of two

distances is equal to the product of the other two distances. The two distances in each product correspond to two people who are opposite each other at the table. **Part B** closest: 13.25 in. farthest: 40.75 in. The closest and farthest distances are on a diameter of the table. Let x be the distance from a person to the salt shaker. The person opposite is $54 - x$. From Part A, $x(54 - x) = 540$. Solving for x gives 13.25 and 40.75.



Topic Review

2. sector of a circle **4.** tangent to a circle **6.** $\frac{28}{9}\pi$ **8.** $\frac{35}{2}\pi$ **10.** From the circumference, you can calculate the radius. Since the area is determined by the radius and central angle, if you know the area and radius, you can set up an equation for the sector area and solve for the central angle. **12.** 32° **14.** 3.9 cm **16.** 9.8 **18.** They are perpendicular. Let X be the midpoint of \overline{AB} . Then $\overline{TX} \perp \overline{AB}$ and $\overline{SX} \perp \overline{AB}$. So, T , X , and S are collinear and $\overline{TS} \perp \overline{AB}$.

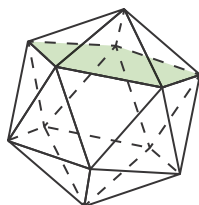


Selected Answers

Topic 11

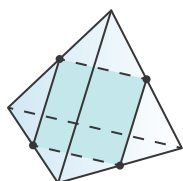
Lesson 11-1

2. Nicholas used his representation of the plane as the cross section. The cross section is a pentagon.



4. 9 **6.** 10 **8.** octagon **10.** $720\pi \text{ cm}^3$

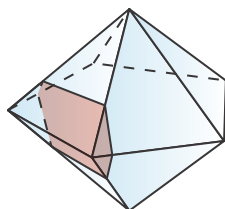
12. Yes; the plane can intersect the midpoints of four edges as shown to form a cross section with four sides.



14. No; think of a cube as a stack of squares. Rotation of points about an axis produces circles, not squares.

16. 14 **18.** 9

20. hexagon



22. an isosceles triangle **24.** a cylinder with a cone on top **26.** 1,800 in.

28. The revolving door is a pair of intersecting rectangles rotating about their line of intersection. The mat will be a circle. **30.** B

Lesson 11-2

2. The diagram does not give the height of either figure, and to apply Cavalieri's Principle, the heights of the two figures must be equal. **4.** $\frac{x^3}{4\pi}$

6. 840 in.^3 **8.** 120 cm^3 **10.** 10.5 m^3

12. Figures B and C; they have the same base area, $\pi \text{ ft}^2$, and the same height, 4 ft. **14.** Yes; the cones have the same radius and height.

The cone on the right is an oblique version of the cone on the left.

16. Yes; $\frac{1}{2}(a)(2b)(x) = \frac{1}{2}(2a)(b)(x)$

18. The original plan called for a pond with a volume of $\pi(3)^2(6)$, or about 169.65 ft^3 . If the depth of the pond changes to 5 ft, $169.65 = \pi r^2(5)$. Solve for r to get $r \approx 3.3$. Talisa should make the radius 3.3 ft. **20.** 1.95 mm

22. 17 bags **24.** Yes; the volume of the box is $(6.5)(3.5)(9.25) \approx 210 \text{ in.}^3$. If the box is 80% full, the volume of cookies needed is $0.8(210) = 168 \text{ in.}^3$. Each cookie has a volume of $\pi(1.5)^2(0.5) \approx 3.5 \text{ in.}^3$. So $168 \div 3.5 = 48$ cookies are needed for each box. If each cookie contains 12 chocolate chips, $48(12) = 576$ chocolate chips are needed for the box. **26.** B

Lesson 11-3

2. Zhang used the formula for the volume of a prism, not the volume of a pyramid. **4.** No; you are given the height, but there is not enough information to compute the radius.

Both are needed to compute the volume. **6.** 66.0 cm^3 **8.** 17.3 m^3

10. $1,771.9 \text{ in.}^3$ **12.** No; the two stacks



Selected Answers

Topic 11

have the same height, but the area of a cross section of the penny stack is smaller than the area of a cross section of the nickel stack. **14.** Answers may vary. Sample: First use the Pythagorean theorem with the slant height and half of a base side to find the altitude. Then the volume is one-third of the square of the base side times the altitude.

16. 229.1 units³ **18.** 339.3 units³

20. 324 units³ **22.** 278.1 units³

24. 1,100 units³ **26.** 52 trucks

28. The volume of a molecule is 0.00043 nm³, so the molecules occupy 2.59×10^{20} nm³. The gas occupies $0.0224 \text{ m}^3 \times \frac{10^{27} \text{ nm}^3}{1 \text{ m}^3} = 2.24 \times 10^{25} \text{ nm}^3$.

The gas is more than 99.99% empty space. **30.** C

Lesson 11-4

2. Reagan reported her answer in square units rather than cubic units. She also wrote the expression for the volume of a sphere correctly, but she wrote S.A. instead of V on the left side of the equation. **4.** $h = \frac{4}{3}r$; $h = 4r$ **6.** 452.4

8. 64.9 **10.** volume: 4.19; surface area: 12.6 **12.** Kayden used the diameter of the sphere where she should have used the radius. **14.** The two stacks have the same volume because they have the same height and equal cross sections at all heights. **16.** $\pi(r - h)^2$

18. $\pi\left(\frac{rh_1}{h + h_1}\right)^2$ **20.** 4,536.5 units²

22. 104.2 in.² **24.** 134.0 units³

26. 65,449.8 m³ **28.** 4,817.1 units³

30. 34 beads **32.** 215.1 cm³ **34.** D

Topic Review

2. oblique **4.** hemispheres **6.** pentagon

8. No; Euler's Law, $F + V = E + 2$, is

true for all convex polyhedrons. If

$F = V = E$, then $F + F = F + 2$, or $F = 2$.

No polyhedron exists with only 2 faces.

10. 226.2 yd³ **12.** The volumes are

equivalent, 2,646 cm³. **14.** 5,654.9 m³

16. about 25.7 in.³ **18.** 1.5 cm³ **20.** 4



Selected Answers

Topic 12

Lesson 12-1

2. The probability is greater if the first marble is not returned to the bag. If the first marble is returned, there will be 6 marbles in the bag, of which 4 are red. If it is not returned, there will only be 5 marbles in the bag, of which 4 are red. $\frac{4}{5} > \frac{4}{6}$. **4.** Events are mutually exclusive if they do not share outcomes. Independent events can share outcomes, but the occurrence of one cannot affect the probability of the other happening. **6.** 44% **8.** $\frac{1}{4}$, 0.25, or 25% **10.** The events C and M are not mutually exclusive, so it is not true that $P(C \text{ or } M) = P(C) + P(M)$. So, subtract the probability that a student is in both clubs to find the probability that a random student is in the Chess Club or the Math Club. $P(C \text{ or } M) = P(C) + P(M) - P(C \text{ and } M)$. **12.** yes **14.** no **16.** 0.5, 50%, or $\frac{1}{2}$; The area of the triangle is half of the area of the rectangle because the rectangle and the triangle have the same base (50 cm) and the same height (40 cm). **18.** 0.68 or 68% **20.** 21% **22.** 32% **24. a.** 0.64, or 64% **b.** 0.04, or 4% **26.** C

Lesson 12-2

2. The sample space for $P(B | A)$ has to take into account that A is necessary. The sample space for $P(B)$ includes all of B , even the parts that do not include A . **4.** From the formula for conditional probability, both $P(A) \cdot P(B | A)$ and $P(B) \cdot P(A | B)$ are equal to $P(A \text{ and } B)$. **6.** No; the coach does not know whether girls and boys at the camp are equally likely to play soccer. **8. a.** dependent; $\frac{1}{90}$ **b.** independent; $\frac{1}{100}$

10. When A and B are independent, $P(B | A) = P(B)$. Then the formula $P(A \text{ and } B) = P(A) \cdot P(B | A)$ simplifies to $P(A \text{ and } B) = P(A) \cdot P(B)$, which is true when events A and B are independent. **12.** Without replacement: $P(\text{yellow second} | \text{blue first}) = \frac{3}{14}$, which is greater than $P(\text{yellow second} | \text{yellow first}) = \frac{2}{14} = \frac{1}{7}$. With replacement: $P(\text{yellow second} | \text{blue first}) = P(\text{yellow second} | \text{yellow first}) = \frac{3}{15} = \frac{1}{5}$. **14.** 0.6 or 60% **16.** about 0.53 or 53% **18.** Dependent; $P(\text{Game Design} | \text{Sophomore}) \approx 53\%$ and $P(\text{Game Design}) = 50\%$, so $P(\text{Game Design} | \text{Sophomore}) \neq P(\text{Game Design})$. **20.** 45% **22.** No; $P(\text{improved} | \text{medication}) \approx 45\%$ while $P(\text{improved} | \text{placebo}) \approx 57\%$. Patients taking the medication showed improvement less frequently than patients taking the placebo. **24.** 0.01 or 1%; Answers may vary. Sample: $P(\text{prize}) = 0.05$ and $P(\text{comic} | \text{prize}) = 0.2$, so $P(\text{prize and comic}) = P(\text{prize}) \cdot P(\text{comic} | \text{prize}) = (0.05)(0.2) = 0.01$

26. 0.3 or 30%; $P(A | \text{defective})$

$$= \frac{P(A \text{ and defective})}{P(\text{defective})} =$$

$$\frac{P(\text{defective} | A) \cdot P(A)}{P(\text{defective})} = \frac{(0.015)(0.2)}{(0.01)} = 0.3$$

28. C

Lesson 12-3

2. The number of combinations of n items chosen r at a time is found by dividing the number of permutations, ${}_nP_r$, by $r!$ **4.** ${}_9C_2$ and ${}_9C_7$ are equivalent; ${}_9C_2 = \frac{9!}{2!7!}$ and ${}_9C_7 = \frac{9!}{7!2!}$, so ${}_9C_2 = {}_9C_7$. In general, ${}_nC_r = \frac{n!}{r!(n-r)!}$



Selected Answers

Topic 12

and ${}_nC_{n-r} = \frac{n!}{(n-r)!r!}$, so ${}_nC_r = {}_nC_{n-r}$.

6. There are ${}_{12}C_4 = 495$ ways to select a committee of 4 people from a group of 12. Only one of these possible committees contains the 4 people newest to the company. $P(4 \text{ newest})$

$$= \frac{\text{choose 4 of 4 newest} \cdot \text{choose 0 of 8 older}}{\text{choose 4 of 12 total employees}}$$

$$= \frac{{}_4C_4 \cdot {}_8C_0}{{}_{12}C_4} = \frac{1}{495}$$
 8. permutation

10. 165 **12.** $\frac{15}{91}$ **14.** $\frac{28}{91}$ **16.** Problem 12 is the same as problem 15 and problem 13 is the same as problem 14. If no prize is awarded to an athlete, then both prizes are awarded to non-athletes. If no prize is awarded to a non-athlete, then both prizes are awarded to athletes. **18. a.** 18,564; the order does not matter, so there are ${}_{18}C_6 = 18,564$ groups of 6 erasers. **b.** 56; there are 8 aliens, so there are ${}_8C_3 = 56$ ways to choose 3 aliens. **c.** 6,720; there are 10 flying saucers, so there are ${}_{10}C_3 = 120$ ways to choose 3 flying saucers. Then multiply by the answer to part b. **d.** 0.36; the probability is the ratio of the number of favorable outcomes (the answer to part c) to the total number of possible outcomes (the answer to part a).

$\frac{6,720}{18,564} \approx 0.36$ **20.** 6; any two points form a line and the order is not important. Because ${}_4C_2 = 6$, 6 lines can be determined by four non-collinear points. **22.** combination; 330

24. combination; 126

26. permutation; 1,320

28. a. $\frac{{}_4P_3 \cdot {}_5P_0}{{}_9P_3} = \frac{24}{504} = \frac{1}{21}$ **b.** $\frac{{}_4P_2 \cdot {}_5P_1}{{}_9P_3}$

$$= \frac{60}{504} = \frac{5}{42}$$
 30. $\frac{9}{28}$; use combinations

because which \$1 bill and which \$10 bill do not matter, and the order in which they are pulled out does not

matter. $\frac{{}_3C_1 \cdot {}_3C_1}{{}_8C_2} = \frac{9}{28}$ **32. a.** $\frac{1}{30,240}$;

order matters, so there are ${}_{10}P_5$ possible codes, only one of which is 30429. So $P(30429) = \frac{1}{{}_{10}P_5} = \frac{1}{30,240}$.

b. $\frac{1}{100,000}$; there are 10^5 possible codes, only one of which is 30429. So $P(30429) = \frac{1}{10^5} = \frac{1}{100,000}$.

34.

	Yes	No
${}_8P_3$	<input type="checkbox"/>	<input checked="" type="checkbox"/>
${}_8C_3$	<input checked="" type="checkbox"/>	<input type="checkbox"/>
$\frac{{}_8P_3}{3!}$	<input checked="" type="checkbox"/>	<input type="checkbox"/>
$8! \cdot 3!$	<input type="checkbox"/>	<input checked="" type="checkbox"/>
$\frac{8!}{3!}$	<input type="checkbox"/>	<input checked="" type="checkbox"/>
$\frac{8!}{5!}$	<input type="checkbox"/>	<input checked="" type="checkbox"/>
$\frac{8!}{3!5!}$	<input checked="" type="checkbox"/>	<input type="checkbox"/>
$8 \cdot 7$	<input checked="" type="checkbox"/>	<input type="checkbox"/>

36. Part A

$$P(V1 \text{ and } V2) = P(V1) \cdot P(V2 | V1)$$

$$= \frac{3}{9} \cdot \frac{2}{8} = \frac{1}{12}$$

$$P(V1 \text{ and } V2) = \frac{{}_3C_2}{{}_9C_2} = \frac{1}{12}$$

Part B $P(\text{SURF}) = \frac{1}{9} \cdot \frac{1}{8} \cdot \frac{2}{7} \cdot \frac{1}{6} = \frac{1}{1,512}$

$$P(\text{SURF}) = \frac{{}_1P_1 \cdot {}_1P_1 \cdot {}_2P_1 \cdot {}_1P_1}{{}_9P_4}$$

$$= \frac{2}{3,024} = \frac{1}{1,512}$$

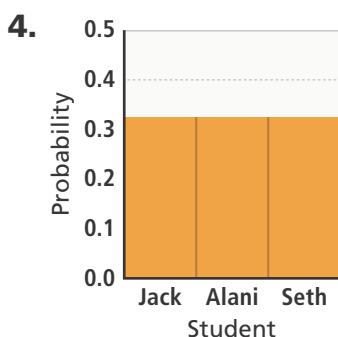


Selected Answers

Topic 12

Lesson 12-4

2. A binomial experiment has specific rules. It can only have two outcomes, the trials must be independent, and the probability for success is the same every trial.



6. 35% **8.** 48% **10.** 41% **12.** The probability distribution is the function P , defined on the set $\{4, 5, 6\}$, such that $P(4) = 0.25$, $P(5) = 0.5$, and $P(6) = 0.25$.

14. Abby forgot to multiply by ${}_7C_5$. $P(5) = {}_7C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^2 = 21 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^2 \approx 0.0384$.

16. Let P be the function defined on the set $\{Y, R, G, B\}$ such that $P(Y) = 0.3$, $P(R) = 0.4$, $P(G) = 0.2$, and $P(B) = 0.1$.

18. Let P be the function defined on the set $\{0, 1, 2, 3, 4\}$ such that $P(0) = \frac{1}{10}$, $P(1) = \frac{7}{30}$, $P(2) = \frac{3}{10}$, $P(3) = \frac{1}{5}$, and $P(4) = \frac{1}{6}$. **20.** Yes; there are 50 trials; each trial has two possible outcomes, success and not success; the performance of one bulb does not affect the performance of another, so the trials are independent; and the probability of success, 0.9, is the same for each trial. **22.** No; the probability of making the free throws is not the same for each trial. **24.** 18.8%

26. 52.6% **28. a.** approximately 56% **b.** Answers may vary. Sample: No; a

sample set of 8 people is too small to make a decision from. In order to make an informed decision, the pharmaceutical company should allow more people to test the new medication. **30.** A, B, C, E

32. Part A. The theoretical probability distribution is the function P defined on the set $\{0, 1, 2, 3, 4, 5\}$ such that $P(0) = 0.03125$, $P(1) = 0.15625$, $P(2) = 0.3125$, $P(3) = 0.3125$, $P(4) = 0.15625$, $P(5) = 0.03125$. (Students may express the probabilities as percents or fractions and they may round decimals/percents.) **Part B.** Answers may vary. Sample: The experimental probability distribution is the function P defined on the set $\{0, 1, 2, 3, 4, 5\}$ such that $P(0) = 0.05$, $P(1) = 0.1$, $P(2) = 0.3$, $P(3) = 0.35$, $P(4) = 0.2$, $P(5) = 0$. **Part C.** The experimental probability distribution is somewhat similar to the theoretical distribution but not identical. A theoretical probability distribution describes the results one would expect if an experiment is repeated many times, and we only did it 20 times.

Lesson 12-5

2. He found the probability of heads, not the expected number of heads. The expected number of heads is $50\% \cdot 10$, or 5. **4.** The class pays an average of \$1.48 in winnings for every lottery ticket sold. **6.** 3.5 **8.** 12 **10.** \$16.90 **12.** Nonrefundable; there is a 20% chance the man will not fly, so there is an 80% chance he will fly. If he purchases the nonrefundable ticket, his cost is \$600 whether he flies or not. If he purchases the refundable ticket, his



Selected Answers

Topic 12

expected cost is $(0.8)(900) + 0.2(0) = \$720$. Based on expected value, he should purchase the nonrefundable ticket.

14. Sample: A mean is an average of known values. Expected value is the mean of values that are unknown but that follow a known or estimated probability distribution. **16.** \$600

18. Option C; the cost for the car owner for Option A is \$900; for Option B, \$820; for Option C, \$750.

20. 27 days **22.** Since 3 of the 4 tosses were Heads, the student concludes that $P(\text{heads})$ is 3 out of 4, or 75%. This is not good reasoning because the sample size is so small. Since a fair coin would have a probability of 50% for Heads, the student should draw a much larger sample before concluding that the probability is not 50%. **24.** \$60,268,000

26. C

Lesson 12-6

2. Generate numbers 1 to 6 on a calculator or using index cards, where each number represents the same number on a number cube. **4.** A fair game requires that participants have an equal chance to win or an expected value of 0, meaning no participant has an advantage. **6.** If a “win” is represented by a positive 1 and a “loss” is represented by a negative 1 and there is an equal chance that a player will win or lose, the sum of +1 and -1 is 0. **8.** (1) Write each friend’s name on a slip of paper, and draw one slip at random from a paper bag to determine who gets the last slice. Each slip has a $\frac{1}{3}$ chance of being chosen. (2) Assign the numbers 1 and 2 to one

friend, 3 and 4 to a second friend, and 5 and 6 to the third friend. Roll a number cube, and give the slice to the friend whose number lands on top. Each person has a $\frac{2}{6}$ chance of being chosen. (3) Assign each friend a number from 0 to 2. Use a calculator to generate a random number from 1 to 9. Divide the number by 3, and give the slice to the friend whose number matches the remainder. Each person has a $\frac{3}{9}$ chance of being chosen.

10. 0.659 or 65.9% **12.** No; there are 2 ways to get 1 head since the sample space is {HH, HT, TH, TT}. **14.** Assign numbers 0 through 7 to each of the candidates, and then generate one of the eight integers randomly three times in a row. If the same number (student) is selected during the second or third trials, ignore the number and generate another number. **16.** Unfair; even numbers are far more likely (27 out of 36 possibilities) than odd numbers.

18. She should keep what she has (and not spin) because the game is not fair and she is more likely to lose money than win. **20. a.** \$19.71 **b.** \$96.00

c. Since $12 \text{ mo} \times \$5.49/\text{mo} = \65.88 , the cost of the insurance for a year is \$65.88. Since the repair costs could be as much as \$1,200 and the probability of a leak could be 8%, the expected costs of a gas leak without insurance could be as high as \$96.00. So I would advise her to buy the insurance.

22. A, B, D **24. Part A** Model 1001: \$48; Model 1002: \$50; Calculate the expected profit after potential lawsuits for each model like this: Model 1001:

$$\$60 - (\$1,200,000)\left(\frac{2}{200,000}\right) = \$48$$



Selected Answers

Topic 12

Model 1002:

$$\$56 - (\$1,200,000)\left(\frac{1}{200,000}\right) = \$50$$

Part B It is recommended that the company stop selling Model 1001 and only sell Model 1002. The company can expect to make more money on Model 1002 than on Model 1001. Also, Model 1002 is a safer tire; fewer people will be injured or die if they have Model 1002 tires than if they have Model 1001 tires.

Topic Review

2. permutation **4.** dependent events **6.** expected value **8.** mutually exclusive **10. a.** 0; The probability he will roll a number that is both even and less than 2 is 0 because these are mutually exclusive events. It is impossible to roll a number that lies in both sets. **b.** $0.\overline{6}$ or $66.\overline{6}\%$; Because the events are mutually exclusive, you can add the probabilities.

$$P(<2) + P(\text{even}) = \frac{1}{6} + \frac{1}{2} = \frac{2}{3} = 0.\overline{6} = 66.\overline{6}\%$$

12. 0.30 **14.** approx. 0.21 **16.** The student's conclusion is correct even though the reasoning is wrong. To test for independence, the student should have compared $P(\text{prime} \mid \text{even})$ with $P(\text{prime})$, or $P(\text{even} \mid \text{prime})$ with $P(\text{even})$. $P(\text{prime} \mid \text{even}) = 0.5$ and $P(\text{prime}) = 0.6$, so the events are dependent. **18.** permutation; 56 **20.** The student computed ${}_5P_2$ instead of ${}_5C_2$. The student needs to divide by $2!$ to complete the calculation. ${}_5C_2 = 10$. **22.** 0.35% **24.** 22.89% **26.** Akasi mixed up the variables n and r . By putting them in the correct place, she can get her solution. $P(3) = {}_5C_3 \cdot 0.24^3 (1 - 0.24)^{5-3} \approx 0.0798 \approx 7.98\%$ **28.** 345 **30.** $x = 67.5$ **32.** No, she should still shoot from the 10-point line. With her increased success rate, her expected points per toss from the 20-point line are $20 \cdot 0.30 = 6$, which is still less than the 6.5 points per toss from the 10-point line.