Selected Answers

Topic 1

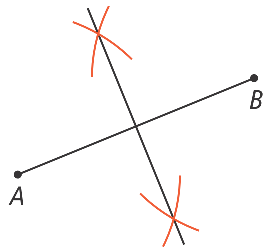
Lesson 1-1

1. The ruler postulate pairs any point on a line with one and only one real number coordinate. The absolute value of the difference of the coordinates connecting two points on a line is the length of the segment connecting those points. The protractor postulate pairs one and only one real number with the measure of the angle between a ray and a point not on the ray.
3. Segments are congruent if they have the same length. Angles are congruent if they have the same measure.
5. 2 **7.** 7 **9.** –1 and 23 **11.** 12 or 52 **13.** 36° **15.** 77° **17.** $2\frac{2}{3}$ **19.** $4\frac{1}{2}$ **21.** $4\frac{5}{6}$ **23.** 10 **25.** 17 **27.** 65° **29.** 3 **31.** 4 **33.** 32° **35.** 12 **37.** Yes; Sample answer: According to the plan, the building will have a height of 185 ft, which is below the height limitation for the area. **39.** A, C, D, F **41.** Students' floor plan should include four rooms, two walls of equal length, and two angles of equal measure, and their equations should show congruent angles and segments.

Lesson 1-2

1. A straightedge is used to make lines or segments. A compass is used to make arcs, measure distances, and mark congruent lengths. Using these tools, you can copy and bisect angles and segments. **3.** A line that is perpendicular to a segment can intersect the segment at any point on the segment. A perpendicular bisector intersects the segment at the point that is the same distance from each endpoint of the segment.

5.



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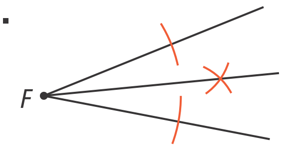
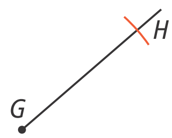


9. at about (–2, 6) **11.** Sample: 2, 4, 8, 16; power of 2 **13.** Check students' work. By aligning *F* and *G*, the segment from *F* to where the crease crosses the segment is the same length as the segment from *F* to where the crease crosses the segment. Also by aligning, the crease is perpendicular to the segment.

15.



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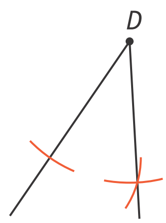

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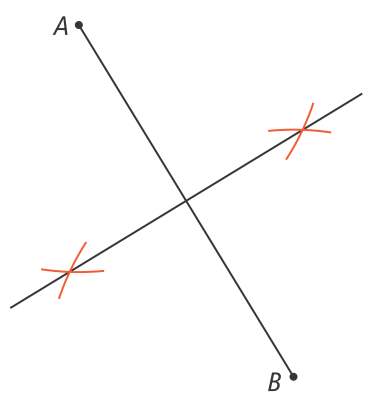
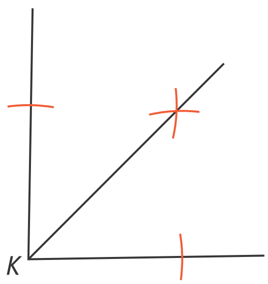


Selected Answers

Topic 1

21.



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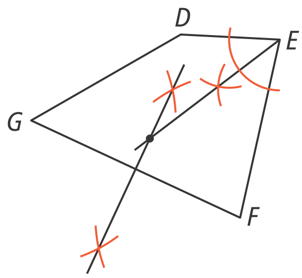
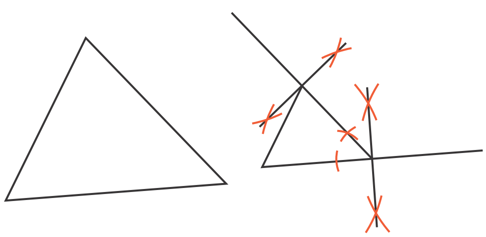


25. 22.5 ft by 30 ft, 675 ft^2; x is width, $1.5x$ is length. $5x = 300$, so $x = 60$. The dimensions of the gym are 60 ft by 90 ft. The width is bisected, so one dimension of each section is 30 ft. The length is bisected to get 45 ft, and then bisected again to get 22.5 ft.
27. $m\angle NPM = 2(m\angle QPM)$
29. Part A Sample:



Part B Check students' work. Bisect an original side. Then copy a segment with the length of one half. Next, copy an adjacent angle. Then make sure the other side of the angle is half the length of the corresponding original side. Repeat until all sides and angles have been copied. **Part C** Bisect a side. Then bisect a half. Copy a segment that has the length of half and a half of a half (so it is three quarters the length of the original side). Copy an adjacent angle. Then make sure the other side of the angle is three quarters the length of the original side. Repeat until all sides and angles have been copied.

Lesson 1-3

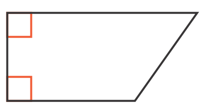
1. The midpoint of a segment is determined by applying the Midpoint formula to the coordinates of the endpoints of the segment. The length of a segment is determined by applying the Distance Formula to the coordinates of the endpoints of the segment. **3.** $PM = MQ$; $PQ = 2PM$
5. $\left(1, -\frac{1}{2}\right)$ **7.** 15 **9. a.** $\left(3, -\frac{11}{2}\right)$
b. $K\left(a + \frac{|a-c|}{4}, b + \frac{|b-d|}{4}\right)$
11. $a = 2$, Sample: Set up simultaneous equations in two variables and solve for a.
$\frac{2a + b + 1}{2} = 3, \frac{a + 3b + 5}{2} = 5 \rightarrow a = 2.$
13. (4, 22) and (11, 15); Using the Pythagorean Triple 8, 15, 17, where 17 is the straight line distance, 8 is the horizontal distance, and 15 is the vertical distance, add 8 to –4 and 15 to 7 to get (4, 22). Switch 8 and 15 to get (11, 15). **15.** (0.8, –3.4) **17.** $\left(\frac{1}{2}, 9\right)$
19. $\left(1\frac{1}{15}, 3\frac{5}{12}\right)$ **21.** Jamie; Arthur is about 45 yards away and Jamie is only about 28 yards away. **23.** $\left(\frac{19}{3}, 5\right)$;
$(1, 9) + \frac{2}{3}[(9, 3) - (1, 9)] = (1, 9) + \frac{2}{3}(8, -6) = \left(\frac{19}{3}, 5\right)$ **25.** Yes; the distance is $\sqrt{24^2 + 44^2} \approx 50.1$ meters. **27.** D

Selected Answers

Topic 1

Lesson 1-4

1. Answers may vary. Sample: By observing cases or patterns, you can make generalizations that could apply to all cases or can be used to predict results. **3.** conjecture **5.** $2n$ **7.** Abby's statement is a conjecture. She has shown that it is true for the first 20 values of n, but she has not proven it for all values of n. **9.** Answers may vary. Sample: In order to prove or disprove this statement, I would look at every whole number between 7,608 and 7,620 and show whether it is prime or composite. If any of the numbers are prime, then a counterexample exists and the statement is false. If none of the numbers are prime, then the statement is true because there are a finite number of cases and all of the cases would have been tested.
11. $\frac{16}{9}, \frac{32}{27}, \frac{64}{81}$ **13.** Each year, about 65% of the senior class have their driver's license. **15.** Answers may vary. Sample:



17. a. Answers may vary. Sample: The data for the four groups is very close. In all four trials, between 20% and 23% of subjects reported better sleep. Therefore, a reasonable conjecture is that the herb is between 20% and 23% effective in improving sleep.
b. Answers may vary. Sample: Between 200 and 230 subjects will report better sleep. **19.** $1 + 2 = 3$; $3 + 4 = 7$; $7 + 6 = 13$; $13 + 8 = 21$; the pattern is to add successive multiples of 2. A design with 6 circles has $21 + 10 = 31$ separate regions. **21.** D **23. Part A** Answers may vary. Sample conjectures: If there is a vote on Proposition 3, then it is likely to pass. If the survey is representative of the population, then Proposition 3 is likely to pass. Sample explanation: In a survey of 300 people, half would be 150. Since more than 150 people are for Proposition 3, it is likely to pass.
Part B Answers may vary. Sample: about 4,000

Lesson 1-5

1. Answers may vary. Sample: Identify the cause and effect. The cause is the hypothesis for a conditional statement and the effect is the conclusion.
3. negation **5.** Answers may vary. Sample: The inverse and contrapositive are alike because the hypothesis and the conclusion are negated. The inverse and contrapositive are different because in the contrapositive, the hypothesis and the conclusion are switched. **7.** Hypothesis: A rectangle has an area of 12 m^2. Conclusion: The rectangle has sides of length 3 m and 4 m. **9.** Converse: If a rectangle has sides of length 3 m and 4 m, then it has an area of 12 m^2; true. Inverse: If a rectangle does not have an area of 12 m^2, then it does not have sides of length 3 m and 4 m; true. Contrapositive: If a rectangle does not have sides of length 3 m and 4 m, then it does not have an area of 12 m^2; false.
11. The hypothesis "20 is a multiple of 3" is false. When a conditional has a false hypothesis, then its truth value is true regardless of the conclusion.

Selected Answers

Topic 1

13. She reversed the hypothesis and conclusion. The correct conditional is, "If water is below 0°C, then it freezes." **15.** No; Sample: If the contrapositive is false, then the inverse is true. **17.** The hypothesis is true and the conclusion is false. **19.** If I cut my hair, then it will be shorter. **21.** If it is Tuesday, then movie tickets are half-price. **23.** False; Sample: $-3 \times -10 = 30$ **25.** Negation of hypothesis: Both whole numbers are odd or both whole numbers are even; Negation of conclusion: The sum of the two numbers is even. **27.** If an angle is not obtuse, then it does not measure 100°. This is true because only obtuse angles are larger than 90° and less than 180°. All other angles are equal to or smaller than 90°. **29.** Answers may vary. Sample: The converse of the conditional is, "If 53% of the dogs are male, then 47% of the dogs are female." This is true because the dogs that are not male are female. Since both the conditional and its converse are true, the biconditional is true. **31.** If two angles are complementary, then their measures add up to 90°. If the measures of two angles add up to 90°, then they are complementary. **33.** Answers may vary. Sample: If I am 65 inches tall in the morning, then I am 64.35 inches tall in the evening. **35. a.** If the day is Monday, then the science museum is closed. **b.** If the day is Thursday, then the science museum is open from 10 a.m. to 8 p.m. **c.** Converse: If the science museum is open from 10 a.m. to 8 p.m., then the day is Thursday. Inverse: If the day is not Thursday, then the science museum is not open from 10 a.m. to 8 p.m. Contrapositive: If the science museum is not open from 10 a.m. to 8 p.m., then the day is not Thursday. The converse is false, the inverse is false, and the contrapositive is true. **d.** The conditional in part (a) can be written as a biconditional but the conditional in part (b) cannot. The converse of part (a) is true, but the converse of part (b) is false. The day is Monday if and only if the science museum is closed. **37.** D

Lesson 1-6

1. Answers may vary. Sample: Inductive reasoning is based on observed patterns. Deductive reasoning is based on logic. **3.** Answers may vary. Sample: Deductive reasoning uses facts and logic to reach a valid conclusion while inductive reasoning looks at patterns to reach a conjecture which may not be proven to be valid. **5.** Casey has a fever; Law of Detachment. **7.** If you eat too much, you want to rest. **9.** Samantha uses the same hypothesis for both conditionals so she can't combine the two using the Law of Syllogism. **11.** Figures A, E **13.** No; for any conditional, if the hypothesis is true, a true or false conclusion results in the conditional being true. **15.** The Law of Detachment cannot be used because the hypothesis of the conditional is not given to be true. **17.** The Law of Detachment cannot be used because the hypothesis of the conditional is not true. **19.** The Law of Syllogism cannot be used because the hypothesis of one conditional is not the conclusion of the other.

Selected Answers

Topic 1

21. If it is Thursday, then he eats grilled chicken for dinner. Charles has baseball practice. He eats grilled chicken for dinner. **23.** p = Avery draws a number card from 4 to 10.
q = He moves his game piece into home base.
r = He wins the game.

If r is false, then $p \rightarrow q$ and $q \rightarrow r$ are false. The contrapositives $\sim r \rightarrow \sim q$ and $q \rightarrow p$ are true; by the Law of Syllogism, the contrapositive $\sim r \rightarrow \sim p$ is true, so by the Law of Detachment, since r is false, p is false. You can conclude that Avery does not draw a number card from 4 to 10. **25.** No; you need to know if someone ate fish and chips or if someone ate tacos. **27.** D

Lesson 1-7

1. A proof starts with a given statement and then includes a series of steps using deductive reasoning to explain why a conjecture is true. **3.** You assume a postulate to be true without proof. A conjecture is a conclusion you reach by inductive reasoning. A theorem is a conjecture that has been proven. **5.** $m\angle ADC = m\angle ADB + m\angle BDC$ **7.** All right angles are congruent. **9.** Def. right angles, Subst. Prop., Def. $\cong$ angles **11.** Because $\angle M$ and $\angle N$ are congruent, $m\angle M = m\angle N$ by the definition congruent angles. Because $\angle M$ and $\angle N$ are supplementary, $m\angle M + m\angle N = 180°$. Substitute $m\angle M$ for $m\angle N$ to get $m\angle M + m\angle M = 180°$, or $2m\angle M = 180°$. Use the Division Property to get $m\angle M = 90°$. Substitute $m\angle N$ for $m\angle M$ to get $m\angle N = 90°$. By the definition of right angles, $\angle M$ and $\angle N$ are right angles. **13.** Since $\angle VZX$ and $\angle WZY$ are right angles, $\angle VZW$ is complementary to $\angle WZX$ and $\angle XZY$ is complementary to $\angle WZX$. By the Congruent Complementary Theorem, $\angle VZW \cong \angle XZY$. **15.** $x = 22$; 109°; 109° **17.** $x = 25$; 125°; 125°

19.

Statements	Reasons
1) $\angle 1$ and $\angle 2$ are compl.	**1)** Given
2) $m\angle 1 + m\angle 2 = 90°$	**2)** Def. compl. $\angle$s
3) $m\angle 1 = 23°$	**3)** Given
4) $23° + m\angle 2 = 90°$	**4)** Subst.
5) $m\angle 2 = 67°$	**5)** Subtr. Prop.
6) $m\angle 2 + m\angle 3 = 180°$	**6)** Linear Pairs Thm.
7) $67° + m\angle 3 = 180°$	**7)** Subst.
8) $m\angle 3 = 113°$	**8)** Subtr. Prop.

21. Division A 97.2°, Division B 82.8°, Division D 64.8°, Division F 46.8°. By Vertical Angles Theorem and the Angle Addition Postulate, the measure of Division C is 97.2° − 64.8° = 32.4°. By the Supplementary Angles Theorem and the Angle Addition Postulate, the measure of Division E is 180° − (32.4° + 64.8° + 46.8°) = 36.0°. Division C is 9%, and Division E is 10%. To find the percentages, multiplying the degree measure by 100 and divide by 360°.
23. a. $\angle 1 \cong \angle 2$; Theorem 1-4
b. $\angle 3 \cong \angle 4$; Vertical Angles Theorem
25. A

Lesson 1-8

1. The initial assumption must be false. **3.** The student should assume $m\angle 2 \leq 30°$. **5.** Given the conditional $p \rightarrow q$, the statement you assume is not q. The statement you try to prove is not p. **7.** Today is not Saturday.

Selected Answers

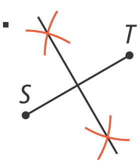
Topic 1

9. Assume $\overline{PQ}$ is not perpendicular to $\overline{ST}$. **11.** Assume $LM \neq 6$. From the diagram, $KL = 6$, so $LM \neq KL$, and by the definition of a perpendicular bisector, $\overline{NJ}$ cannot intersect $\overline{KM}$ at its midpoint. Therefore $\overline{NJ}$ cannot be the perpendicular bisector of $\overline{KM}$. This contradicts the original statement, so the assumption must be wrong. Therefore $LM = 6$. **13.** Contrapositive: If $x = y$, then $\angle TUV$ and $\angle WXY$ are not congruent. Assume $x = y$. Given $m\angle TUV$, substitute $x = y$ to get $y^2 + xy = x^2 + x^2 = 2x^2$. Given $m\angle WXY$, substitute $x = y$ to get $3x(x + y) = 3x(x + x) = 6x^2$. Since $2x^2 \neq 6x^2$, you can conclude that $\angle TUV$ and $\angle WXY$ are not congruent, which proves the contrapositive. Therefore the conditional statement is true: If $\angle TUV \cong \angle WXY$, then $x \neq y$. **15.** Paige is not in Mr. Green's 2nd period class. **17.** $ST + TU + UV \neq 150$ **19.** I and III **21.** Assume $x = 3y$. Then $x + y = 3y + y = 4y$. Also, $2x - y = 2(3y) - y = 5y$. Since $\angle EFG$ and $\angle HFJ$ are vertical angles, $x + y$ must equal $2x - y$, which implies $4y$ must equal $5y$. But $4y = 5y$ only if $y = 0$, and y cannot equal 0 if $x + y = 2x - y$. The assumption is therefore false, so $x \neq 3y$. **23. a.** $x^2 + h^2 = d^2$ **b.** Contrapositive: If d does not decrease, then x does not decrease. Assume d does not decrease. Then either d remains the same or d increases. Suppose d remains the same. Since $x^2 = d^2 - h^2$, and since h is a constant, $d^2 - h^2$ does not change, so x does not change. Suppose d increases. Then since $x^2 = d^2 - h^2$, and since h is a constant, $d^2 - h^2$ increases, so x increases. In either case, x does not decrease, which proves the contrapositive. Therefore the original conditional is true: If x decreases, then d decreases. **25.** Assume Nicky does not have less than 2 miles left. Then she has 2 miles or more to go. The distance from home to the library is 0.4 mile + 1.2 miles = 1.6 miles. Since the library is the midpoint, the distance from library to the museum is also 1.6 miles. This contradicts the statement that Nicky has 2 miles or more to go, so the assumption is wrong. Therefore Nicky has less than 2 miles left to go. **27.** D

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** inductive reasoning **5.** Law of Detachment **7.** contrapositive **9.** 6 **11.** 20° **13.** 56°

15.



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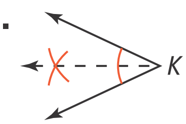


19. The arcs will not intersect if the compass width is less than half of the segment width. **21.** $\left(-1, \frac{5}{2}\right)$; $\sqrt{17}$ **23.** (3, 0); $2\sqrt{5}$ **25.** $\left(-\frac{3}{5}, \frac{1}{5}\right)$ **27.** 720, 5040 **29.** 27, 35 **31.** counterexample: right triangle **33.** A true statement must always be true, so one counterexample shows it is not true. An example only shows that the statement is true in at least one case, but there may be other cases where it is not true. **35.** Conditional: If a number is a multiple of 4, then it is a multiple of 2. Converse: If a number is a multiple of 2, then it is a multiple of 4.

Selected Answers

Topic 1

Inverse: If a number is not a multiple of 4, then it is not a multiple of 2. Contrapositive: If a number is not a multiple of 2, then it is not a multiple of 4. **37.** False; 1 is less than 4, but it is not prime. **39.** false; counterexample: a day with such little snowfall that school is not cancelled **41.** $DE = 12$ **43.** You have fresher breath. **45.** $x = 37$; 106°; 106°

47. Contrapositive: If $x = 12$, then $GJ \neq 48$. Assume $x = 12$. By the Segment Addition Postulate, $GJ = GH + HJ$, and by substitution $GJ = 2x + x = 3x$. Substitute $x = 12$. Then $GJ = 3(12) = 36$. Therefore $GJ \neq 48$. Since this proves the contrapositive, the original conditional must be true, so $x \neq 12$.

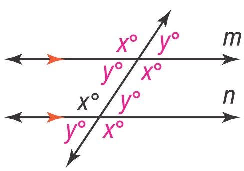
Selected Answers

Topic 2

Lesson 2-1

1. Corresponding, alternate interior, and alternate exterior angles are congruent. Same-side interior and same-side exterior angles are supplementary. The two types of congruent angles formed are supplementary to each other.
3. Answers may vary. Sample: Leah cannot use the theorem because it is not indicated that the lines intersected by the transversal are parallel.
5. congruent, Corresponding Angles Theorem **7.** 71° **9.** 68° **11.** $\angle 1$ and $\angle 3$ are supplementary and $\angle 2$ and $\angle 4$ are supplementary because they are same-side interior angles.

13.



15. Answers may vary. Sample: $\angle 4$ and $\angle 7$ **17.** Answers may vary. Sample: $\angle 6$ and $\angle 3$ **19.** 123°

21.

Statements	Reasons
$\overline{AB} \parallel \overline{CD}$	Definition of parallelogram
$m\angle B + m\angle C = 180°$ and $m\angle A + m\angle D = 180°$	Same-Side Interior Angles Postulate
$\overline{BC} \parallel \overline{AD}$	Definition of parallelogram
$m\angle A + m\angle B = 180°$ and $m\angle C + m\angle D = 180°$	Same-Side Interior Angles Postulate
$m\angle A + m\angle B = m\angle B + m\angle C$	Trans. Prop. of Eq.
$m\angle A = m\angle C$	Subtr. Prop. of Eq.
$\angle A \cong \angle C$	Def. of congruence
$m\angle B + m\angle C = m\angle C + m\angle D$	Trans. Prop. of Eq.
$m\angle B = m\angle D$	Subtr. Prop. of Eq.
$\angle B \cong \angle D$	Def. of congruence

23. $y = 50$ because that angle and the labeled 50° angle are corresponding angles. The angles marked x and y are same-side interior angles, so $x + y = 180$ and $x = 130$. **25. a.** 50° **b.** corresponding angles; congruent **27.** congruent to $\angle 1$: $\angle 6$, $\angle 8$, $\angle 3$; congruent to $\angle 2$: $\angle 5$, $\angle 7$, $\angle 4$
29. Part A the northwest corner of Skyline Trail and Mission Path and the northeast corner of Skyline Trail and Hood Path **Part B** Trail Marker 131

Selected Answers

Topic 2

Lesson 2-2

1. corresponding angles, alternate interior angles, same-side interior angles, alternate exterior angles **3.** It has a theorem, property, postulate, or definition in each box, with a reason below each box. Arrows show the order of the logical steps from the given statement to the conclusion. **5.** Converse of the Alternate Interior Angles Theorem **7.** $m\angle 1 + m\angle 3 = 180°$ or $m\angle 2 + m\angle 4 = 180°$ **9.** $\angle 2$ and $\angle 3$ are not corresponding angles. **11.** Sample: Draw an angle bisector between any opposite vertices of the hexagon so that it forms a transversal between opposite sides of the hexagon. By the Converse of the Opposite Interior Angles Theorem, the lines containing opposite sides are parallel, so the opposite sides are parallel. **13.** $p \parallel q$; Converse of the Alternate Interior Angles Theorem **15.** $r \parallel s$; Converse of the Same-Side Interior Angles Theorem

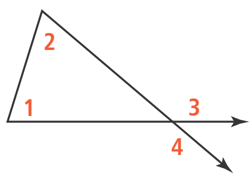
17. $a \parallel c,\ b \parallel c$ (Given) → $\angle 1 \cong \angle 3,\ \angle 2 \cong \angle 3$ (Corr. $\angle$s Thm.) → $\angle 1 \cong \angle 2$ (Trans. Prop.) → $a \parallel b$ (Converse of the Corr. $\angle$s Thm.)

19. 7; Converse of the Alternate Exterior Angles Theorem **21.** 70°; Converse of the Alternate Exterior Angles Theorem **23.** supplementary, congruent **25. Part A** $\angle 2 = 125°$ using the Converse of the Corresponding Angles Theorem.; $\angle 3 = 55°$ by the Converse of the Same-Side Interior Angles Theorem. **Part B** Sample: Using Theorem 2-9, if the angle formed by each aisle is perpendicular to a line on the floor, then the aisles are parallel.

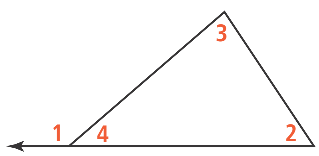
Lesson 2-3

1. The sum of the measures of the interior angles in a triangle is equal to 180 degrees. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles in the triangle. **3.** Remote interior angles are distant from an external angle. **5.** 35 **7.** 103 **9.** $m\angle 1 + m\angle 2 + m\angle 3 = 180°$; $m\angle 4 = m\angle 1 + m\angle 2$

11. Yes; in the diagram shown, $\angle 1$ and $\angle 2$ are remote interior angles for both $\angle 3$ and $\angle 4$.



13. Given:



Prove: $m\angle 1 = m\angle 2 + m\angle 3$

$m\angle 2 + m\angle 3 + m\angle 4 = 180°$, Triangle Angle-Sum Theorem

$m\angle 1 + m\angle 4 = 180°$, Linear Pairs Theorem

Selected Answers

Topic 2

$m\angle 1 + m\angle 4 = m\angle 2 + m\angle 3 + m\angle 4$, Trans. Prop. Eq.
$m\angle 1 = m\angle 2 + m\angle 3$, Subtraction Property of Equality

15. $5x + x = 180$ or $x + 2x + 3x = 180$; $30°, 60°, 90°, 150°$ **17.** $x = 48$ **19.** $x = 20$; $y = 50$ **21.** 121 **23.** 51 **25.** 79° **27.** 138° **29. a.** $w° + x° = 90°$ **b.** $z° = 90° + y°$ **31. a.** 49° **b.** 33° **c.** It increases to 122°. **33.** C

Lesson 2-4

1. If the slopes of two lines are equal, the slopes are parallel. If the product of the slopes is –1, the lines are perpendicular. Any two vertical lines are parallel, and any vertical line is perpendicular to any horizontal line. $\left(-\frac{2}{3}\right)\left(-\frac{3}{2}\right) = 1$ **3.** $x = 4$; No, any vertical line is perpendicular to $y = 0$. **5.** When computing the slope of $\overleftrightarrow{CD}$, he didn't use the same order for coordinates in the numerator and denominator. The slope of $\overleftrightarrow{CD}$ is $\frac{-1-(-4)}{1-5} = -\frac{3}{4}$. **7.** no **9.** yes **11.** $y = -\frac{1}{3}x - 5$ **13.** q: $y = \frac{5}{3}x + \frac{8}{3}$; m: $y = -\frac{3}{5}x + \frac{24}{5}$ **15.** k: $y = -\frac{5}{4}x + \frac{7}{2}$, j: $y = \frac{4}{5}x + \frac{38}{5}$ **17.** A rectangle has two pairs of sides that are parallel, and adjacent sides are perpendicular. None of the lines have equal slope, so there are no parallel lines. **19.** The slope of $f(x)$ is 2 and the slope of $g(x)$ is 5; the lines are not parallel. **21.** no **23.** yes **25.** yes **27.** $y = \frac{3}{2}x + \frac{5}{2}$; $y = -\frac{2}{3}x + \frac{1}{3}$ **29.** No; slope of slide 1 $= \frac{-72}{40} = -\frac{9}{5}$; slope of slide 2 $= \frac{-40}{24} = -\frac{5}{3}$; Since the slopes are not equal, the slides are not parallel. **31.** C, F **33. Part A** 1, –1 **Part B** Answers may vary. Sample: $y = x - 1$, $y = -x + 14$, $y = x + 2$, $y = -x + 10$ **Part C** 3

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** transversal **5.** corresponding angles **7.** same-side interior angles **9.** 16 **11.** 56° **13.** 49° **15.** 142° **17.** no; slope of w: $m_w = \frac{6-4}{8-2} = \frac{1}{3}$; slope of t: $m_t = \frac{0-5}{7-5} = -\frac{5}{2}$; $m_w m_t = \left(\frac{1}{3}\right)\left(\frac{-5}{2}\right) = -\frac{5}{6}$ **19.** parallel: $y = -3x + 13$; perpendicular: $y = \frac{1}{3}x + \frac{19}{3}$

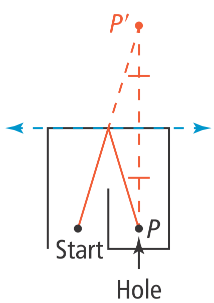
Selected Answers

Topic 3

Lesson 3-1

1. The reflected image of a point is on the line through the preimage perpendicular to the line of reflection. The point and its image are equidistant to the line of reflection. **3.** A rigid motion is a transformation that produces an image with both angle measures and side measures that are equal to those of the preimage, so the measures are not changed and the lines of the figure are not bent. **5.** Draw R' as the image of the reflection of the object across the surface. Draw segment from target to R' and place a dot at the intersection with the surface. From that dot, draw a segment to R. Draw the path from the object to the intersection point and then from the intersection point to the target. **7.** $(-5, -3)$ **9. a.** $R_t(x, y) \rightarrow (-8 - x, y)$, where t is the line $x = -4$ **b.** $R_t(x, y) \rightarrow (x, -14 - y)$, where t is the line $y = -7$ **11. a.** $A'(6, 1)$, $B'(9, 5)$, $C'(4, 10)$; $A''(0, 1)$, $B''(-3, 5)$, $C''(2, 10)$ **b.** $R_t(x, y) \rightarrow (x, 8 - y)$, where t is the line $y = 4$; $R_q(x, y) \rightarrow (6 - x, y)$, where q is the line $x = 3$

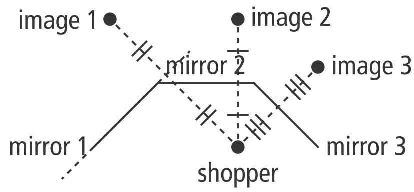
13. Jacob is incorrect. He should reflect the Hole P over the back wall at P'. Then connect the Start point with P'. Jacob should aim at the intersection of the back wall with this second line.



15. $y = x - 1$ **17.** Yes; all corresponding angles and sides appear to be the same size. **19.** $A'(9, 3)$, $B'(6, -4)$, $C'(-1, 5)$

21. $A'(-19, -3)$, $B'(-16, 4)$, $C'(-9, -5)$
23. $A'(-3, 9)$, $B'(4, 6)$, $C'(-5, -1)$
25. $R_d(x, y) \rightarrow (4 - x, y)$ where d is the line $x = 2$ **27.** $R_d(x, y) \rightarrow (x, -10 - y)$ where d is the line $y = -5$

29.

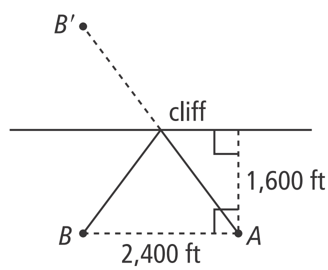


31.

Yes; all lengths and angle measures are the same.

33. rigid motion, not a rigid motion, rigid motion, not a rigid motion

35. Part A



Part B

$2{,}400 \text{ ft} \div 1{,}000 \text{ ft/s} = 2.4 \text{ s}$

$2\left(\sqrt{1600^2 + 1200^2}\right) = 4{,}000 \text{ ft}$

$4{,}000 \text{ ft} \div 1{,}000 \text{ ft/s} = 4 \text{ s}$

Selected Answers

Topic 3

Lesson 3-2

1. Translation preserves length and angle measures but changes the location. **3.** Answers may vary. Sample: $(T_{\langle 3,\,-2\rangle} \circ R_{x\text{-axis}})\,(\triangle PQR)$. **5.** $X(-3, -7)$, $Y(-6, -4)$, $Z(-1, -2)$ **7.** $T_{\langle 7,\,-3\rangle}(ABCD) = A'B'C'D'$ **9.** $T_{\langle 4,\,9\rangle}$ **11.** Hugo did the reflection before the translation. **13.** Answers may vary. Sample: $T_{\langle 1,\,3\rangle} \circ T_{\langle 4,\,0\rangle} \circ T_{\langle 2,\,3\rangle}$ **15.** $A'(8, -1)$, $B'(2, 1)$, $C'(9, -4)$ **17.** $G'(-10, -5)$, $H'(-7, 1)$, $J'(2, -6)$ **19.** Answers may vary. Sample: $T_{\langle 8,\,0\rangle} \circ T_{\langle 0,\,-5\rangle}$ **21.** $T_{\langle -5,\,7\rangle}$ **23.** $T_{\langle -2,\,-1\rangle}$ **25.** $T_{\langle -2,\,0\rangle}$ **27.** $T_{\langle 0,\,-4\rangle}$ **29.** 12 **31.** The sheriff drives 40 miles east and 50 miles south from Coby to Dallinger. Then she drives 20 miles west and 10 miles north to Lotan.

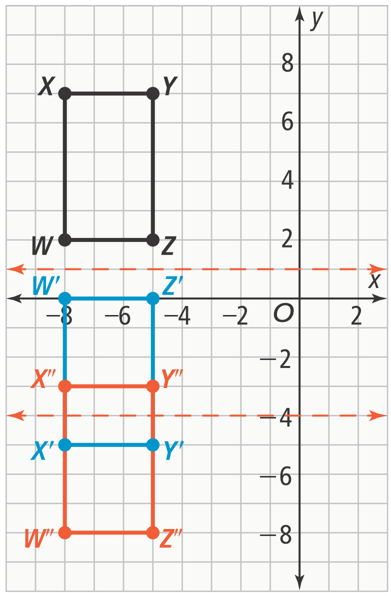
33.

	Yes	No
$T_{\langle 0,\,10\rangle}(\triangle ABC)$	☐	☑
$T_{\langle 10,\,0\rangle}(\triangle ABC)$	☑	☐
$(R_{y\text{-axis}} \circ R_a)(\triangle ABC)$	☐	☑
$(R_b \circ R_c)(\triangle ABC)$	☑	☐

35. Answers may vary. Sample:

Part A

$(R_{x=-4} \circ R_{x=1})\,(WXYZ)$



Part B

$(R_m \circ R_n)\,(WXYZ) = (T_{\langle 0,\,-10\rangle})\,(WXYZ)$, where m is the line $y = 1$ and n is the line $y = -4$. The lines of reflection $y = 1$ and $y = -4$ are parallel. By Theorem 3-1, any translation is equivalent to reflection over two parallel lines.

Part C

Perimeter $WXYZ = 16$ units

Perimeter $W''X''Y''Z'' = 16$ units

Area $WXYZ = 15$ square units

Area $W''X''Y''Z'' = 15$ square units

Translation has no effect on either the perimeter or area. Perimeter and area are calculated from side lengths, and translation preserves side lengths.

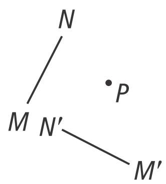
Selected Answers

Topic 3

Lesson 3-3

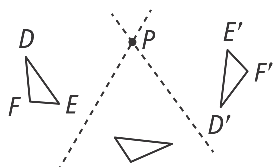
1. The distance from each point to the center of rotation is the same as the distance from the corresponding image to the center of rotation. The angles formed by each point, the center of rotation, and the corresponding image all have a measure equal to the angle of rotation. **3.** A rotation of a point by 360° forms a circle. The center of the circle is the center of rotation.

5.



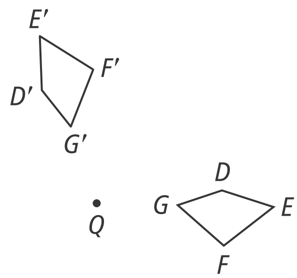
7. $G'(-2, 9)$, $H'(1, -3)$

9.

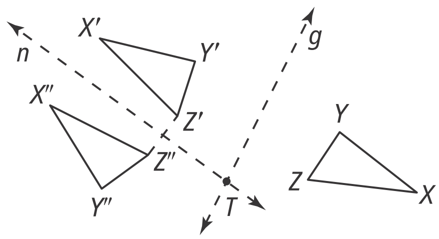


11. No; the angle of rotation is the same, and each point and its image are the same distance from the center of rotation, but the distance each point moves varies. **13.** $\frac{1}{3}$; If the line is rotated 270°, the rotated line is perpendicular to the original line. Perpendicular slopes are opposite reciprocals. **15.** Answers may vary. Sample: Rotate $\triangle ABC$ 180° about point C, then translate 4 units down and 8 units left.

17.



19. $D'(-5, 0)$, $E'(-8, -2)$, $F'(5, -3)$
21. $S'(2, 6)$, $T'(5, -3)$, $U'(-1, 0)$
23. Answers may vary. Sample:



25. 45° counterclockwise; If Gear A rotates 90°, it turns through 2 of the teeth on the gear. This corresponds to 2 teeth on the driver gear, which has 16 teeth total. $\frac{2}{16} = \frac{x}{360}$, so $x = 45$.
27. The pattern repeats five times around the circle with no gaps or overlaps, so the angle of rotation is $360° \div 5 = 72°$. Rotate each triangle 72° counterclockwise using A as the center of rotation. Complete the design by rotating the image of each rotation three more times using the same center and angle of rotation. **29.** B

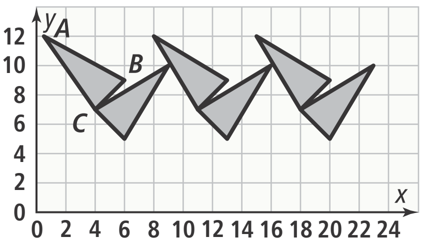
Lesson 3-4

1. reflection, translation, rotation, glide reflection **3.** The transformation shows a reflection followed by a rotation. It should be a reflection followed by a translation. **5.** translation **7.** rotation
9. He translated *ABCD* 6 units right instead of reflecting it over the

Selected Answers

Topic 3

line $x = 0$. **11.** $A'(12, 3)$, $B'(12, 7)$, $C'(10, 5)$ **13.** $A'(-6, -6)$, $B'(-3, 6)$, $C'(7, 7)$ **15.** $A'(-12, 4)$, $B'(-9, 16)$, $C'(1, 17)$ **17.** $A'(13, 0)$, $B'(10, -12)$, $C'(0, -13)$ **19.** Answers may vary. Sample: $(T_{\langle 2, 0\rangle} \circ R_t)(\triangle DEF) = \triangle D'E'F'$, where t is the line $y = 8$ **21.** Answers may vary. Sample: $(T_{\langle -6, 0\rangle} \circ R_{x\text{-axis}})(\triangle DEF) = \triangle D'E'F'$ **23.** $(T_{\langle 5, 0\rangle} \circ R_t)(\square ABCD) = \square A'B'C'D'$, where t is the line $y = 7$ **25.** I. C II. B III. A IV. D **27. Part A** and **Part B** $(T_{\langle 0, -2\rangle} \circ R_t)(\triangle ABC)$ where t is the line $x = 12$; $(T_{\langle 0, -2\rangle} \circ R_p)(\triangle ABC)$ where p is the line $x = 19$



Lesson 3-5

1. Figures can have reflectional symmetry, rotational symmetry, and point symmetry. Reflectional symmetry can be identified by finding lines that can be drawn through the center that create halves that are mirror images. Rotational symmetry can be identified by finding rotations about a point in the center that map a figure onto itself. If a figure has 180° rotational symmetry, then it also has point symmetry. **3.** reflectional symmetry **5.** Yes, a non-rectangle parallelogram has 180° rotational symmetry but no reflectional symmetry. **7.** 1 **9.** 120° and 240°; the figure does not have point symmetry **11.** reflectional symmetry; 8; rotational symmetry; 45°, 90°, 135°, 180°, 225°, 270°, 315° **13.** Answer may vary. Sample: Rotational symmetry occurs through angles of rotation that put a long petal that is behind the other long petals at the top position. Since there are 3 such petals and 360° ÷ 3 = 120°, the angles occur at multiples of 120°. **15.** 180° and 270° rotational symmetry **17.** Answers may vary. Sample: decorative patterns like those found on floors, carpets, and moldings **19.** reflection over a vertical line through the center, reflection over a horizontal line through the center, 180° rotation **21.** 45°, 90°, 135°, 180°, 225°, 270°, and 315° **23.** Three lines of reflection and rotations at 120° and 240° **25.** You could visualize folding each flag along a line of reflection and rotating it about its center. **27. a.** Six lines of reflection; rotational symmetry at 60°, 120°, 180°, 240°, and 300°; point symmetry **b.** vertical line of reflection **c.** rotational symmetry at 180°, point symmetry **29.** D

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** rigid motion **5.** glide reflection **7.** image **9.** $H'(3, 2)$, $J'(1, -3)$, $K'(-4, 3)$ **11.** The slope of the line through the two points should be the negative reciprocal of the slope of the line, and the midpoint of the two points is a point on the line. **13.** $P'(0, -2)$, $Q'(2, -2)$, $R'(5, -8)$ **15.** The distance of the translation will be twice the difference between the lines $x = 2$ and $x = -3$, so the distance is 10 units. **17.** $A'(-2, -2)$, $B'(-2, 3)$, $C'(3, 1)$

Selected Answers

Topic 3

19. $r_{(180°,\ P)}$ **21.** $L'(0, -11)$, $M'(3, -9)$, $N'(-1, -2)$ **23.** Yes; the translation and reflection can be performed in either order. **25.** no reflectional symmetry; 180° rotational symmetry about the center of the figure **27.** The figure must have rotational symmetry because the composition of the reflections across two lines of symmetry is a rotation about the center of the figure that will map the figure on to itself.

Selected Answers

Topic 4

Lesson 4-1

1. Two figures are congruent if there is a composition of rigid motions that maps one figure to the other. **3.** A rigid motion maps a figure to a congruent figure. **5.** Yes; since each intermediate image is mapped from the original figure by rigid motions, each intermediate figure is congruent to the original. Then by the Transitive Property of Congruence, the intermediate figures are congruent to the final figure. **7.** 180° rotation about (0, 5) **9.** Answers may vary. Sample: translation to the right, then reflection over a horizontal line **11.** no; Given $AB = 5$ and $CD = 6$, you can translate point A to map it to point C, but there is no composition of rigid motions that will then map point B to point D. **13.** yes; Given any two rays $\overrightarrow{AB}$ and $\overrightarrow{CD}$, translate $\overrightarrow{CD}$ to map endpoint C to endpoint A. Then rotate $\overrightarrow{CD}$ around point C until $\overrightarrow{CD}$ maps to $\overrightarrow{AB}$. Since each ray continues infinitely in one direction, the remaining points on $\overrightarrow{CD}$ map to the remaining points on $\overrightarrow{AB}$. **15.** 90° counterclockwise rotation around point W **17.** $A \cong C$; reflection over a vertical line **19.** Yes; the right shoe is a reflection of the left shoe over a vertical line. **21.** congruent; rigid motion

23. Part A

Part B upper left: is a rotation about the midpoint between two corresponding vertices; all others except purple figure: glide reflections **Part C** Purple figure is not congruent. It has the same orientation and shape, but is not the same size as the original image.

Lesson 4-2

1. Isosceles triangles have two congruent sides, and the angles opposite those sides are congruent. Equilateral triangles have three congruent sides, and the measure of each angle is 60°. **3.** The base of an isosceles triangle is between the two congruent angles. The legs are opposite the two congruent angles. **5.** Rotate 120° about the center of the triangle. Rotate 240° about the center of the triangle. Reflect across the line through vertex P and the midpoint of $\overline{QR}$. Reflect across the line through vertex Q and the midpoint of $\overline{PR}$. Reflect across the line through vertex R and the midpoint of $\overline{PQ}$.
7. $m\angle D = 51.5°$; $m\angle F = 51.5°$
9. $KL = 26$; $LM = 26$; $MK = 20$
11. 6.6 ft **13.** By the Isosceles Triangle Theorem, $\angle QPS \cong \angle QRS$, so $m\angle QPS = m\angle QRS$. Reflecting $\triangle PQS$ horizontally across $\overline{QS}$ shows that $PS = SR$. It also shows that $m\angle PSQ = m\angle RSQ$. By the Linear Pairs Theorem, $m\angle PSQ + m\angle RSQ = 180°$, so each angle must equal 90°. Therefore, $\overline{QS} \perp \overline{PR}$.
15. For an isosceles triangle, the legs are congruent, but the base is not necessarily congruent. In this case, $2x$ is

Selected Answers

Topic 4

not equal to 6. Instead $2x = 3x - 5$, so $x = 5$. The length of each leg is 10.
17. Reflect $\triangle JKL$ horizontally right across a line perpendicular to $\overline{JL}$ to form image $\triangle L'K'J'$. This shows that $JK \cong J'K'$. Then translate $\triangle L'K'J'$ left to form image $\triangle L''K''J''$ so that $\overline{L''K''}$ maps onto $\overline{JK}$. This shows that $J'K' \cong KL$. By the Transitive Property, $JK \cong KL$.
19. $m\angle S = 56°$; $m\angle U = 56°$
21. $GH = 45$, $HJ = 45$, $GJ = 29$
23. $16\sqrt{2}$ **25.** 32° **27. a.** 5.8 ft
b. The tent is equilateral and therefore also isosceles. The perpendicular bisector of the bottom of the tent is also the altitude, so a right triangle is formed and the length can be found with the Pythagorean Theorem.
29. a. $5y - 19 = 56$ **b.** 68°
31. Part A 17.7 cm **Part B** 21.3 cm

Lesson 4-3

1. SAS: If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. SSS: If all side lengths of one triangle are congruent to all side lengths of another triangle, then the triangles are congruent. **3.** No; those are not corresponding sides of $\triangle PQR$ and $\triangle STU$. To use *SAS*, two sides and an included angle of $\triangle PQR$ must be congruent to two sides and an included angle of $\triangle STU$. **5.** Yes; since the segments bisect each other, $JP = KP$ and $LP = MP$. The vertical angles at point P are congruent, so the triangles are congruent by SAS. **7.** neither
9. Yes; $\angle ACB$ and $\angle DCE$ are vertical angles, so $m\angle ACB = m\angle DCE$. By the Triangle Angle-Sum Theorem, $m\angle B = 180 - m\angle A - m\angle ACB$ and $m\angle E = 180 - m\angle D - m\angle DCE$, so $\angle B \cong \angle E$. $\triangle ABC \cong \triangle DEC$ by SAS.
11. Yes; since $\angle T \cong \angle W$ by Alternate Interior Angles, and these angles lie between congruent sides, the triangles are congruent by SAS.
13. Given: $\triangle ABC \cong \triangle XYZ$.
Prove: $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, $\overline{AC} \cong \overline{XZ}$, $\angle A \cong \angle X$, $\angle B \cong \angle Y$, and $\angle C \cong \angle Z$.
Let $AB = r$, $BC = s$, $AC = t$, $m\angle A = a$, $m\angle B = b$, and $m\angle C = c$. If $\triangle ABC \cong \triangle XYZ$, If then there is a sequence of rigid motions that maps A to X, B to Y, and C to Z. Rigid motions preserve distance, so $AB = XY = r$, $BC = YZ = s$, and $AC = XZ = t$. Rigid motions also preserve angle measures, so $m\angle A = m\angle X = a$, $m\angle B = m\angle Y = b$, and $m\angle C = m\angle Z = c$. Therefore, $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, $\overline{AC} \cong \overline{XZ}$, $\angle A \cong \angle X$, $\angle B \cong \angle Y$, and $\angle C \cong \angle Z$. **15.** $\triangle ABC \cong \triangle DEF$ by SSS and $\triangle DEF \cong \triangle GHJ$ by SAS, $\triangle ABC \cong \triangle GHJ$ by the Transitive Property of Congruence.
17. Given: $ABCDE$, where $\angle EAB \cong \angle ABC \cong \angle BCD \cong \angle CDE \cong \angle DEA$, and $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE} \cong \overline{EA}$
Prove: $\triangle ACE$ is an isosceles triangle

Statements	Reasons
$\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\angle ABC \cong \angle EDC$	Given
$\triangle ABC \cong \triangle EDC$	SAS
$\overline{AC} \cong \overline{EC}$	CPCTC
$\triangle ACE$ is an isosceles triangle	Definition of isosceles

19. $\overline{RP}$ and $\overline{UT}$ **21.** Given $\overline{RS} \cong \overline{TU}$, $\overline{RV} \cong \overline{VU}$, and $\angle TSV \cong \angle STV$. $\triangle STV$ is isosceles by the definition of isosceles

Selected Answers

Topic 4

triangles. $\overline{SV} \cong \overline{TV}$ by the Converse of the Isosceles Triangle Theorem. So $\triangle RSV \cong \triangle UTV$ by SSS. **23.** No; the players run the same distance. Given that the field is a rectangle, the opposite sides that measure 10 m are congruent. By the Reflexive Property of Congruence, the south side of the field is congruent to itself and the southwest and southeast corners are congruent because they are right angles. So, by SAS there are congruent triangles and the diagonals run by the girls are congruent by CPCTC. **25.** The information is not sufficient to say whether the scarves are congruent. You need to know that all three side lengths are congruent, or that the included angles are congruent to determine if the scarves are congruent.
27. D

Lesson 4-4

1. ASA: If two angles of one triangle and the included side are congruent to two angles and the included side of another triangle, then the two triangles are congruent. AAS: If two angles and a non-included side of one triangle are congruent to two angles and a non-included side of another triangle, then the two triangles are congruent.
3. AAS **5.** $x = 3$ **7.** congruent by AAS
9. Using the given information, you can show that $\triangle LMN \cong \triangle PQR$ and $\triangle LNO \cong \triangle PRS$ by SAS. Two polygons composed of congruent triangles are congruent.
11. Given: $\overleftrightarrow{WZ} \parallel \overleftrightarrow{XY}$, $\overleftrightarrow{WX} \parallel \overleftrightarrow{ZY}$
Prove: $\overline{WX} \cong \overline{YZ}$

Statement	Reasons
$\overleftrightarrow{WZ} \parallel \overleftrightarrow{XY}$, $\overleftrightarrow{WX} \parallel \overleftrightarrow{ZY}$	Given
$\angle WXZ \cong \angle YZX$	Alternate interior angles are congruent if the lines are parallel.
$\angle WZX \cong \angle YXZ$	Alternate interior angles are congruent if the lines are parallel.
$\overline{XZ} \cong \overline{XZ}$	Reflexive Prop. of Congr.
$\triangle WXZ \cong \triangle YZX$	ASA
$\overline{WX} \cong \overline{YZ}$	CPCTC

13. Acceptable answers: $\overline{JK} \cong \overline{NO}$, $\overline{JL} \cong \overline{NM}$, or $\overline{KL} \cong \overline{OM}$ **15.** All right angles are congruent. The 28° angles are given as congruent. The vertical beam is congruent to itself. The two right triangles are congruent by ASA. The slanted beams are congruent by CPCTC.

17. Given: $\angle A \cong \angle C$, $\overline{BX} \cong \overline{DX}$
Prove: $\overline{AX} \cong \overline{CX}$

Statement	Reasons
$\angle A \cong \angle C$, $\overline{BX} \cong \overline{DX}$	Given
$\angle AXD \cong \angle BXC$	Vertical Angles Theorem
$\triangle AXD \cong \triangle CXB$	AAS
$\overline{AX} \cong \overline{CX}$	CPCTC

19. Yes; because $ABCD \cong EFGH$ there exists a sequence of rigid motions that maps $ABCD$ to $EFGH$. Since isometries preserve side lengths, $\overline{AB} \cong \overline{EF}$, $\overline{BC} \cong \overline{FG}$, $\overline{CD} \cong \overline{GH}$, and $\overline{AD} \cong \overline{EH}$. Isometries also preserve angle measures, so $\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, and $\angle D \cong \angle H$.

Selected Answers

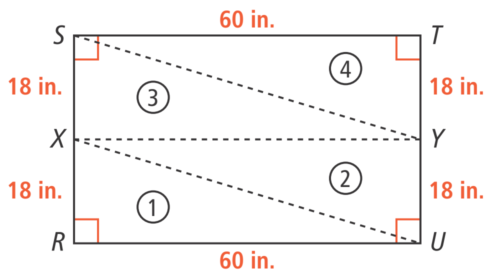
Topic 4

21. Answers may vary. Sample: Given $AB = BD$. $\angle C \cong \angle E$ and $\angle A \cong \angle D$ because they are alternate interior angles. $\triangle BAC \cong \triangle BDE$ by AAS.

23.

$\angle W$	$\angle Y$
$\angle XZW$	$\angle ZXY$
$\angle WXZ$	$\angle YZX$
$\overline{ZX}$	$\overline{XZ}$
$\overline{WZ}$	$\overline{YX}$

25. Part A



Part B Let X and Y be the midpoints of the 36-inch sides. Show that $\triangle 1 \cong \triangle 2$, then $\triangle 1 \cong \triangle 3$, and then $\triangle 3 \cong \triangle 4$. If $\triangle 3 \cong \triangle 4$, then $\triangle 1 \cong \triangle 4$. If $\triangle 1$ is congruent to $\triangle 2$, $\triangle 3$, and $\triangle 4$, then they are all congruent to each other. **Part C** He could make a cut that bisects $\overline{ST}$ and $\overline{RU}$. Then he could cut each of those halves along a diagonal to make four flags.

Lesson 4-5

1. For right triangles, you can show congruence if one pair of angles and one pair of sides are congruent or if two pairs of sides are congruent.
3. The two triangles must be right triangles. They must have at least one pair of congruent legs. They must have congruent hypotenuses.
5. $\angle KLM$ is a right angle. **7.** either $DE = 28$ or $AC = 12$ **9.** Check students' work. **11.** The proof shows congruence of the hypotenuse of one triangle and a leg of another triangle. The HL Theorem requires congruence of corresponding legs of two triangles and corresponding hypotenuses of the triangles. **13.** $\angle C \cong \angle F$ **15.** $\overline{AC} \cong \overline{EF}$
17. $GL = 9$ and $HK = 15$ **19.** Since $\overline{AC} \perp \overline{DB}$, $\angle ADB$ and $\angle BDC$ are right angles. $\overline{BD} \cong \overline{BD}$ by the Reflexive Property. Therefore, given $\overline{AB} \cong \overline{CB}$, $\triangle ABD \cong \triangle BCD$ by the HL Theorem.
21. Measure $\overline{KN}$ and $\overline{MN}$ to confirm that they are congruent. **23.** Measure $\overline{PQ}$ and $\overline{TU}$ to confirm that the hypotenuses are congruent. Then measure $\overline{PR}$ and $\overline{VU}$ to show they are congruent, or measure $\overline{QR}$ and $\overline{TV}$ to show that they are congruent. **25.** D

Lesson 4-6

1. All congruence theorems can be used to prove overlapping triangles congruent: AAS, ASA, SAS, SSS, and HL.
3. In order to use SAS triangle congruence, the congruent angle must be between the congruent sides. In this diagram, the congruent angle is not between the congruent sides. Yes; you can prove that the triangles are congruent.

Given:
$\overline{EF} \cong \overline{DG}$
$\angle F \cong \angle G$

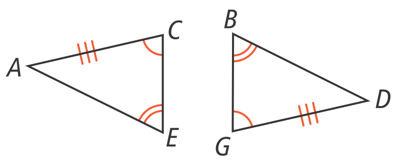
Answers may vary. Sample: Let H be the point where $\overline{FD}$ intersects $\overline{GE}$. $\angle FHE \cong \angle GHD$ because they are vertical angles. $\triangle EFH \cong \triangle DGH$ by AAS.

Selected Answers

Topic 4

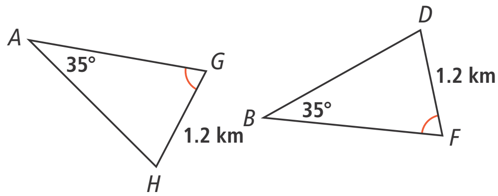
$\overline{HF} \cong \overline{HG}$ and $\overline{HE} \cong \overline{HD}$ by CPCPC. (Note: From here there are several different directions the proof can take.) $\overline{DF} \cong \overline{EG}$ using the Segment Addition Postulate and properties of congruence. $\triangle EFD \cong \triangle DGE$ by SAS. **5.** Corresponding sides: $\overline{WX}$ and $\overline{XW}$, $\overline{WV}$ and $\overline{XY}$, $\overline{VX}$ and $\overline{YW}$; Corresponding angles: $\angle VWX$ and $\angle YXW$, $\angle WXV$ and $\angle XWY$, $\angle XVW$ and $\angle WYX$ **7.** $\overline{AC}$ **9.** $\overline{GK}$ **11.** HL **13.** Since $\overline{AC} \parallel \overline{GD}$, $\angle BAF \cong \angle DGE$. Similarly, since $\overline{CE} \parallel \overline{FB}$, $\angle BFA \cong \angle DEG$. So since $\overline{AF} \cong \overline{EG}$, $\triangle ABF \cong \triangle GDE$ by ASA. **15.** $\triangle ABC$, $\triangle BCD$, $\triangle CDE$, $\triangle DEF$, $\triangle EFA$, $\triangle FAB$; $\triangle BDF$, $\triangle ACE$; $\square ABDE$, $\square BCEF$, $\square CDFA$; $\square ABCE$, $\square BCDF$, $\square CDEA$ **17.** $\angle FHL$ **19.** $\angle XVZ$ **21. Given:** $\overline{NS} \cong \overline{OS}$, $\overline{MN} \cong \overline{PO}$, $\angle M \cong \angle P$ **Prove:** $\triangle MRO \cong \triangle PQN$ **Proof:** Since $\overline{NS} \cong \overline{OS}$, $\triangle NOS$ is isosceles and therefore $\angle ONS \cong \angle NOS$. Because $\overline{MN} \cong \overline{PO}$ and $\overline{NO} \cong \overline{NO}$, by the Segment Addition Postulate, $\overline{MO} \cong \overline{PN}$. Also, it is given that $\angle M \cong \angle P$, so by ASA, $\triangle MRO \cong \triangle PQN$.

23.



25. Yes; by SAS, $\triangle RST \cong \triangle QUT$, and therefore $\overline{ST} \cong \overline{UT}$. So, $\overline{RP} \cong \overline{QM}$, and by ASA, $\triangle RNP \cong \triangle QNM$, which means that $\overline{MN} \cong \overline{PN}$. **27.** B, C, D **29. Part A** She can run the triangular route represented by $\triangle HAG$. Use the Pythagorean Theorem to find that $DG = DF = 1.2$ km. Since $\triangle DGF$ is isosceles, $\angle DGF \cong \angle DFG$. Also $\angle AGH \cong \angle DGF$ because they are vertical angles, so $\angle AGH \cong \angle DFB$. So, by AAS, $\triangle AGH \cong \triangle BFD$, so the perimeters are congruent.

Part B



Part C No; there is not enough information to determine the missing side lengths.

Topic Review

1. You can use any of the triangle congruence criteria: SSS, SAS, ASA, AAS, or HL. • Triangles are congruent if the corresponding angles and sides are congruent or if the corresponding angles and sides can be mapped from one triangle to another by a rigid motion. • If two pairs of corresponding sides and the pair of included angles are congruent, then two triangles are congruent. • If all three pairs of corresponding sides are congruent, the triangles are congruent. • If two pairs of corresponding angles and one pair of corresponding sides are congruent, the triangles are congruent. • Right triangles are congruent if one pair of legs and the hypotenuses are congruent. **3.** base **5.** vertex **7.** No; the two figures are not the same size. **9.** 45°, 90° **11.** 30°, 60°, 90° **13.** There is not enough information to tell if these two triangles are congruent. There are two pairs corresponding congruent sides but the pair of congruent angles is not the pair of included angles.

Selected Answers

Topic 4

15. SSS **17.** Yes; if the pair of congruent sides is included by the two pairs of congruent angles, then the triangles are congruent by ASA. If the pair of congruent sides is not included, then the triangles are congruent by AAS. **19.** Both triangles are right triangles by definition. It is given that the hypotenuses are congruent. Each triangle has a leg with a common segment and another segment that is given to be congruent to each other. By the Segment Addition Postulate, the triangles have a leg with the same length, so there is a pair of congruent legs. The two triangles are congruent by HL. **21.** Both triangles have a right angle and all right angles are congruent. The length of one triangle's hypotenuse is given as $3x$, and the length of the other triangle's hypotenuse is $x + x + x = 3x$ by the Segment Addition Postulate. Since the lengths of the hypotenuses are equal, the hypotenuses are congruent by definition. It is given that the two triangles have a congruent pair of angles. By AAS, the triangles are congruent. **23.** Place point E on $\overline{AC}$ such that $AE = CD$. By applying the Segment Addition Postulate and the Substitution Property, $AD = AE + ED = CD + ED = CE$ so $\overline{AD} \cong \overline{CE}$. By the Converse of the Isosceles Triangle Theorem, $\overline{AB} \cong \overline{BC}$. Then using SAS, $\triangle ABD \cong \triangle CBE$.

Selected Answers

Topic 5

Lesson 5-1

1. The endpoints of a segment are equidistant from any point on its perpendicular bisector. The sides of an angle are equidistant from any point on its bisector. **3.** Answers may vary. Sample: The diagram does not indicate that $\overline{LM} \perp \overline{KL}$ and $\overline{MJ} \perp \overline{KJ}$. **5.** 10 **7.** 36° **9. Given:** $\angle BAD \cong \angle CAD$, $\overline{BD} \perp \overline{AB}$, $\overline{CD} \perp \overline{AC}$ **Prove:** $BD = CD$

Statement	Reason
1) $\angle BAD \cong \angle CAD$	1) Given
2) $\overline{BD} \perp \overline{AB}$, $\overline{CD} \perp \overline{AC}$	2) Given
3) $\angle ABD$ and $\angle ACD$ are right angles.	3) Def. of perpendicular
4) $\angle ABD \cong \angle ACD$	4) Right angles are congruent.
5) $\overline{AD} \cong \overline{AD}$	5) Reflexive Prop. of Congruence
6) $\triangle ABD \cong \triangle ACD$	6) AAS
7) $\overline{BD} \cong \overline{CD}$	7) CPCTC
8) $BD = CD$	8) Def. of congruent segments

11. Yes. Sample: By the Converse of the Angle Bisector Theorem, $\angle ABF \cong \angle CBF$. By the Isosceles Triangle Theorem, $\angle BAF \cong \angle BCF$. By ASA, $\triangle BAF \cong \triangle BCF$, so $\overline{AF} \cong \overline{FC}$ by CPCTC. Therefore, by HL, the triangles are congruent. **13.** Draw acute angle A. Put the compass point on vertex A. Draw an arc that intersects the sides of $\angle A$. Label the points of intersection B and C. Put the compass point on point C and draw an arc. With the same compass setting, draw an arc from point B. Be sure the two arcs intersect. Label the point where the two arcs intersect as D. Draw $\overrightarrow{AD}$. $\overrightarrow{AD}$ is the bisector of $\angle CAB$. Check students' diagrams and constructions. **15.** 3.5 **17.** 6.6 **19.** 8.1 **21. Given:** $\angle ACF \cong \angle ECF$, $\angle ABF$ and $\angle EDF$ are right angles. **Prove:** $\triangle ABF \cong \triangle EDF$

Statement	Reason
1) $\angle ACF \cong \angle ECF$	1) Given
2) $\angle ABF$ and $\angle EDF$ are right angles.	2) Given
3) $BF = DF$	3) Angle Bis. Thm.
4) $\overline{BF} \cong \overline{DF}$	4) Def. of congruent segments
5) $\angle ABF \cong \angle EDF$	5) Right angles are congruent.
6) $\angle BFA \cong \angle DFE$	6) Vertical Angles Thm.
7) $\triangle ABF \cong \triangle EDF$	7) ASA

23. 23°

Selected Answers

Topic 5

25.

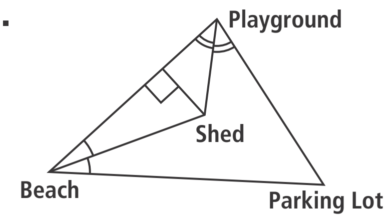
	Yes	No
$AP = XP$	☐	☑
$AB = XY$	☐	☑
$AP = BP$	☐	☑
$XB = YB$	☑	☐
$AY = XB$	☐	☑
$XP = YP$	☑	☐

27. Part A Sample: Measure $\overline{ED}$ and $\overline{FD}$. Then use the right angle to check that $\angle AED$ and $\angle CFD$ are right angles. If $ED = FD$ and if $\angle AED$ and $\angle CFD$ are right angles, then $\angle ABD \cong \angle CBD$ by the Converse of the Angle Bisector Theorem. **Part B** Sample: Use the steel square to check if $\angle BDA$ is a right angle. Then measure to see whether $AB = BC$ or $AD = DC$. If it is, then all three congruencies are met. If $\angle BDA$ is a right angle and $AD = DC$, then $AB = BC$ by the Perpendicular Bisector Theorem, and $\angle ABD \cong \angle CBD$ by CPCTC since $\triangle ABD \cong \triangle CBD$ by SSS. If $\angle BDA$ is a right angle and $AD = DC$, then by the Converse of the Perpendicular Bisector Theorem, B lies on the perpendicular bisector of $\overline{AC}$ and $\overline{BD}$ must be the perpendicular bisector. So $AD = DC$ by the definition of bisector, and $\angle ABD \cong \angle CBD$ by CPCTC since $\triangle ABD \cong \triangle CBD$ by SSS.

Lesson 5-2

1. The perpendicular bisectors intersect at a point that is equidistant from the vertices. The angle bisectors intersect at a point that is equidistant from the sides. **3.** The circumcenter is equidistant from the vertices. The incenter is equidistant from the sides. **5.** 6 **7.** 5 **9.** 8.2 **11.** Sample answer: $\overline{XY}$ is vertical with midpoint (0, a). Its perpendicular bisector is a horizontal line with equation $y = a$. $\overline{XZ}$ is a horizontal line with midpoint (b, 0). Its perpendicular bisector is a vertical line with equation $x = b$. The bisectors intersect at the point (b, a), which is also the midpoint of $\overline{YZ}$. Since $\overline{XY}$ is vertical and $\overline{XZ}$ is horizontal, $\angle X$ is a right angle and $\triangle XYZ$ is a right triangle. Therefore, $\overline{YZ}$ is the hypotenuse of the triangle. The diameter of a circle circumscribed about a right triangle is equal to the length of the hypotenuse. **13.** $\frac{1}{2}a(x + y + z)$ **15.** $\triangle JTL$, $\triangle JTK$, $\triangle KTL$ **17.** P **19.** 9 **21.** 13

23.



The point of intersection of the angle bisectors is equidistant from the sides, which represent the paths. So the shed should be built at this point.
25. Yes; the radius of the maximum ball size that will fit is the distance from the incenter to a side, which is $\sqrt{9^2 - 7.8^2} \approx 4.5$ cm. Since the ball has a radius of 4 cm, it will fit. **27.** B

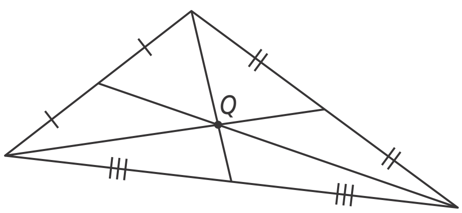
Lesson 5-3

1. The medians of a triangle have a point of concurrency called the centroid. The distance from a vertex to

Selected Answers

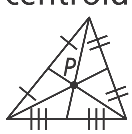
Topic 5

the centroid is two-thirds the length of the median with the vertex as an endpoint. The altitudes of a triangle have a point of concurrency called the orthocenter. **3.** Each segment in the triangle is an angle bisector drawn to the opposite side. Therefore *P* is an incenter, not a centroid. **5.** The centroid is always inside a triangle whether it is acute, right, or obtuse. The orthocenter is inside an acute triangle, at the vertex of the right angle of a right triangle, and outside an obtuse triangle.
7. $AF = 27$; $BD = 36$; $CE = 24$
9. centroid at *Q*



11. circumcenter and centroid; incenter; circumcenter and orthocenter **13.** The centroid is always inside a triangle. The circumcenter is inside an acute triangle, on the midpoint of the hypotenuse of a right triangle, and outside of an acute triangle. A triangle with a common centroid and circumcenter would have the same segments for medians and for perpendicular bisectors and thus be equilateral. **15. a.** angle bis. **b.** median **c.** altitude **d.** perp. bis.

17. centroid at *P*



19. a. (13.5, 3) **b.** (10, 8) **21.** (17.11, 44) **23.** mean, altitude, centroid **25. Part A** Answers may vary. Sample: Each side length is a number between 4 cm and 8 cm and can form a triangle. The point where the wire is attached is the centroid. Sample: side lengths 8 cm, 5 cm, and $\sqrt{41}$ cm; vertices (0, 0), (8, 0), and (5, 4); centroid $\left(4\frac{1}{3}, 1\frac{1}{3}\right)$.
Part B Yes; the triangle must be equilateral. Answers may vary. Sample: side lengths 4 cm, 4 cm, 4 cm

Lesson 5-4

1. When comparing two sides, the longer side is always opposite the larger angle and vice versa. Knowing the relative sizes of the angles means that the sides across from those angles have the same relationship. In any triangle, the sum of the lengths of any two sides is greater than the length of the third side. **3.** Without knowing the length of side $\overline{XZ}$, he cannot be certain that $\angle X$ is the largest angle.
5. $m\angle A < m\angle C < m\angle B$ **7.** $\overline{QR}$ **9.** yes
11. no **13. 3)** Angle Addition Postulate **4)** Comparison Property of Inequality **5)** Substitution Property **7)** Comparison Property of Inequality **8)** Transitive Property of Inequality **15.** Answers may vary. Sample: The sum of the two known sides is 7 ft. In order for the segments to make a triangle, the third side must be less than the sum of the other two sides and longer than the difference of the other two sides.
A correct statement is that the third side could be less than 7 ft. and greater than 1 ft. **17.** $\angle L$; $\overline{JK}$ is the shortest side **19.** $\angle G$ **21.** $\overline{NP}$ **23.** no **25.** yes
27. between 5 ft and 15 ft

Selected Answers

Topic 5

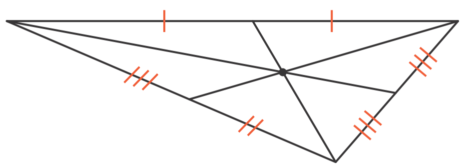
29. between 60 km and 240 km
31. between \$664.65 and \$822.90
33. B **35.** D

Lesson 5-5

1. The shorter third side is opposite the smaller included angle, and the longer third side is opposite the larger included angle. **3.** Only the measure of the included angle varies directly with the length of the third side of a triangle given that the other side lengths remain unchanged. **5.** *CD, AB, BC* **7.** Kayak A **9** Construct an angle with vertex at point *A* with measure equal to $m\angle YWX$. Then locate point *D* on the side of the angle such that $AD = WY$, so $\overline{AD} \cong \overline{WY}$. $\angle YWX \cong \angle DAB$ by definition and $\overline{WX} \cong \overline{AB}$ is given, so $\triangle WXY \cong \triangle ABD$ by SAS. Construct the angle bisector of $\angle CAD$. Let point *E* be the point where the angle bisector intersects $\overline{BD}$. The definition of angle bisector tells you that $\angle DAE \cong \angle CAE$. Applying the Transitive Property of Congruency, $\overline{AD} \cong \overline{AC}$, and by the Reflexive Property of Congruency $\overline{AE} \cong \overline{AE}$. Using SAS, $\triangle ADE \cong \triangle ACE$. Then $\overline{CE} \cong \overline{DE}$ by CPCTC. The Segment Addition Postulate tells you that $DB = DE + EB$ and by the Substitution Property, $DB = CE + EB$. Applying the Triangle Inequality Theorem on $\triangle BCE$, $CE + EB > BC$, so $DB > BC$. Since $\triangle WXY \cong \triangle ABD$, $\overline{YX} \cong \overline{DB}$ by CPCTC, and you can substitute *YX* for *DB*, so $XY > BC$.
11. a. $x < 25$ **b.** $x < 20$ **13.** Airplane B; the 300 mile distance and the 200 mile distance airplane A flies forms two sides of a triangle with an included angle of 165°. The 300 mile distance and the 200 mile distance airplane B flies forms two sides of a triangle with an included angle of 160°. By the Hinge Theorem, the distance from the airport to airplane A is longer than the distance from the airport to airplane B because $165° > 160°$, so airplane B is closer to the airport.
15. Start to *A* to *D* to Finish **17.** B, D **19. Part A** Danielle **Part B** any direction between 37° south of west to 27° east of north

Topic Review

1. The sides and angles of a triangle have bisectors that are concurrent at specific points. The longest side of a triangle is opposite the largest angle. If two triangles have two congruent sides, the longer third side is opposite the larger included angle. **3.** concurrent
5. altitude **7.** median **9.** 52 **11.** 17
13. *K* **15.** 20° **17.** If the circumcenter is on a side of the triangle, the triangle is a right triangle, and the circumcenter must be at the midpoint of the hypotenuse of the right triangle. The area of the circle is πr^2, where *r* is half the length of the hypotenuse. **19.** $\overline{EX}$
21. $\left(\frac{23}{3}, \frac{20}{3}\right)$
23.



25. yes **27.** no **29.** $\angle R$ **31.** between \$52.65 and \$427.05 **33.** $x > 13$

Selected Answers

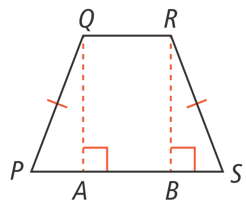
Topic 6

Lesson 6-1

1. The sum of the measures of the exterior angles of a convex *n*-gon is 360° for all values of *n*. The sum of the measures of the interior angles of a convex *n*-gon is $180° \bullet (n - 2)$. **3.** the number of sides **5.** 1,260° **7.** 100° **9.** $m\angle 1 = 125°$, $m\angle 2 = 30°$, $m\angle 3 = 50°$, $m\angle 4 = 95°$, $m\angle 5 = 60°$ **11.** From Example 1, a polygon with *n* sides can be divided into $n - 2$ triangles. By the Triangle Angle-Sum Theorem, the sum of the measures of the angles in a triangle is 180°. Therefore, the sum of the measures of the angles of a polygon is the number of triangles, $(n - 2)$, multiplied by the sum of the measures of the angles in each triangle, 180°. **13.** Find the number of sides of a regular polygon with interior angles measuring 40° by solving the equation $\frac{180° \bullet (n-2)}{n} = 40°$, which results in $n = \frac{360}{140} \approx 2.57$. Since *n* is not a whole number, a regular polygon cannot have an interior angle that measures 40°. **15.** The measure of each exterior angle is 180° minus the adjacent interior angle measure. There are *n* exterior angles. The sum *S* of the exterior angles of a polygon with *n* sides is equal to $n(180°)$ minus the sum of the interior angle measures. Using the formula for the sum of the interior angles measures, $S = n(180°) - (180° \bullet (n - 2)) = 180° \bullet n - 180° \bullet n + 360° = 360°$. **17.** 1440°; 144° **19.** 18 **21.** 6 **23.** $x = 50$; four 120° interior angles and four 150° interior angles **25.** 80° **27.** No, the exterior angle at point *A* is 255°. Each camera must have a field of view of 127.5°. **29.** B

Lesson 6-2

1. In a kite, angles formed by non-congruent sides are congruent and the diagonals are perpendicular. In an isosceles trapezoid, base angles are congruent and diagonals are congruent. **3.** Point *X* and point *Y* are the midpoints of the legs of the trapezoid. **5.** 90° **7.** 24 mm **9.** 21 ft **11.** 43.5 cm
13. Given: Isosceles trapezoid *PQRS*.
Prove: $\angle QPS \cong \angle RSP$



Given that $\overline{PQ} \cong \overline{SR}$ and $\overline{PS} \parallel \overline{QR}$, draw altitudes $\overline{QA}$ and $\overline{RB}$. The altitudes are congruent by the definition of the distance between parallel lines. By the HL Theorem, $\triangle PQA \cong \triangle SRB$. By CPCTC, $\angle QPS \cong \angle RSP$. **15. Given:** Kite *JKLM*, diagonal $\overline{KM}$, $JK < JM$, $KL < LM$
Prove: $\angle JMK \cong \angle LMK$

Statements	Reasons
1) *JKLM* is a kite	**1)** Given
2) $JK < JM$, $KL < LM$	**2)** Given
3) $\overline{KJ} \cong \overline{KL}$, $\overline{MJ} \cong \overline{ML}$	**3)** Def. of a kite
4) $\overline{KM} \cong \overline{KM}$	**4)** Refl. Prop. of ≅
5) $\triangle KJM \cong \triangle KLM$	**5)** SSS
6) $\angle JMK \cong \angle LMK$	**6)** CPCTC

Selected Answers

Topic 6

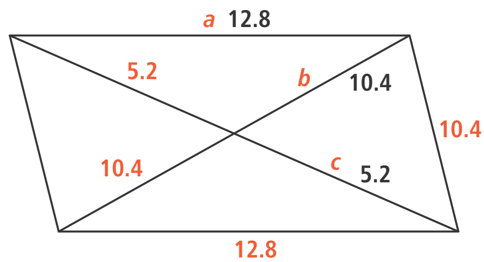
17. 63° **19.** 102° **21.** Yes; The area of the kite is the sum of the areas of two isosceles triangles. Both triangles have a base that is 30 in. Use the Pythagorean Theorem to find the heights: 8 in. and 36 in. The total area of the kite is 660 in.^{2}. **23.** 78°; The seat and pipe form an isosceles trapezoid. The upper base angles are 102° so the lower base angles must be supplementary to 102°. **25.** E

Lesson 6-3

1. In a parallelogram, opposite sides are congruent, consecutive angles are supplementary, opposite angles are congruent, and diagonals bisect each other. **3.** Divide *DF* by 2; Answers may vary. Sample: Point *K* is the intersection of the diagonals of a parallelogram, so it is the midpoint of each diagonal. Therefore, *DF* is one-half of *DK*. **5.** 7 **7.** 99° **9.** 5 **11.** 56° **13.** Construct $\overline{XZ}$, two points define a segment; $\angle WXZ \cong \angle YZX$, Alternate Interior Angles Theorem; $\angle XZY \cong \angle ZXY$, Alternate Interior Angles Theorem; $\overline{XZ} \cong \overline{ZX}$, Reflexive Property of Congruence; $\triangle WXZ \cong \triangle YZX$, ASA; $\overline{WX} \cong \overline{YZ}$, $\overline{WZ} \cong \overline{YX}$, CPCTC. **15.** $\angle A$ is supplementary to $\angle B$, $\angle B$ is supplementary to $\angle C$, $\angle C$ is supplementary to $\angle D$, and $\angle D$ is supplementary to $\angle A$ by the Same-Side Interior Angles Postulate; $m\angle A + m\angle B = 180°$, $m\angle B + m\angle C = 180°$, $m\angle C + m\angle D = 180°$, and $m\angle D + m\angle A = 180°$ by the definition of supplementary. **17.** The diagonals intersect at (3, 4). The diagonals of a parallelogram bisect each other, so they intersect at their midpoints. **19.** 63° **21.** 95°

23. $m\angle A + m\angle B = 180°$, $m\angle B + m\angle C = 180°$, $m\angle C + m\angle D = 180°$, and $m\angle D + m\angle A = 180°$ by Theorem 6-8; $m\angle A + m\angle B = m\angle B + m\angle C$ and $m\angle B + m\angle C = m\angle C + m\angle D$ by the Substitution Property of Equality; $m\angle A = m\angle C$ and $m\angle B = m\angle D$ by the Subtraction Property of Equality; $\angle A \cong \angle C$ and $\angle B \cong \angle D$ by the definition of congruence. **25.** The new bars are parallel to the other bars, so *ABCD* is divided into three parallelograms. Since opposite sides of a parallelogram are congruent, each of the bars must have the same length.

27.



29. Part A 60°; Pipe A makes a 90° angle against Pipe B. Because there are 360° around a point, the obtuse angle(s) in Pipe B must be 360° − 90° − 120° = 150°. Consecutive angles in a parallelogram are supplementary, so the acute angle(s) in Pipe B must be 180° − 150° = 30°. The edge of pipe B is perpendicular to the ground, so the acute angle of Pipe B and the value of $x°$ must be complementary, so $x° = 90° - 30° = 60°$. **Part B** 3 inches; Pipes A and B are joined, so one side of Pipe B must be 3 inches. Since Pipe B is a parallelogram, the opposite side must also be 3 inches. **Part C** $y = 30$, $z = 150$; The measures of the opposite angles were computed in Part A. Since Pipe B is a parallelogram, opposite angles are congruent.

Selected Answers

Topic 6

Lesson 6-4

1. A quadrilateral is a parallelogram if one of the following conditions exists:

Two pairs of opposite sides are congruent.
One pair of opposite sides is congruent and parallel.
Both diagonals bisect each other.
One angle is supplementary to both consecutive angles.
Two pairs of opposite angles are congruent.

3. Yes; quadrilateral *WXYZ* is a parallelogram because two pairs of opposite angles are congruent. **5.** 65° **7.** 14 **9.** There is no relationship between the lengths of the diagonals in a parallelogram. The diagonals bisect each other, but each diagonal can have a different length. **11.** In order to apply Theorem 6-15, the same pair of opposite sides must be both congruent and parallel. If one pair of opposite sides is congruent and the other pair of sides is parallel, the figure could be an isosceles trapezoid. **13.** Apply Theorem 6-12, because all angles measure 90°, and 90° + 90° = 180°, so consecutive angles are supplementary. Or, apply Theorem 6-13, because all angles measure 90°, so both pairs of opposite angles are congruent. **15.** Rotate $\triangle PQR$ about the midpoint of $\overline{QR}$ by 180°. The resulting quadrilateral $PQP'R$ is a parallelogram because opposite sides $\overline{PQ}$ and $\overline{P'R}$ (or $\overline{P'Q'}$) are congruent, and opposite sides $\overline{PR}$ and $\overline{P'Q'}$ (or $\overline{P'R'}$) are congruent. Similarly, rotate $\triangle PQR$ about the midpoint of $\overline{PR}$ and $\overline{PQ}$. **17. a.** $x = -0.5$, $y = 5$ **b.** $x = 17$, $y = 26$ **19.** Yes; given that the top side is parallel to the bottom side, the left and right sides are transversals intersecting parallel lines. Same-side interior angles are supplementary, so the two unknown interior angles are equal to 180° – 132° = 48°. Consecutive angles are supplementary, and opposite angles are congruent, so the figure must be a parallelogram by Theorem 6-12 or Theorem 6-13.
21. Yes; regardless of how far the lamp is extended or retracted, the lengths of each side of each quadrilateral section remain the same. Since two pairs of opposite sides are congruent, the figure is a parallelogram. **23.** The two lines are parallel and of equal length. A quadrilateral with a pair of opposite sides that are congruent and parallel is a parallelogram. **25.** D

Lesson 6-5

1. Rectangles and squares have congruent diagonals. Rhombuses and squares have diagonals that are perpendicular and that bisect the angles of the figure. **3.** Yes; Answers may vary. Sample: If opposite sides of a quadrilateral are congruent, it is a parallelogram, and a parallelogram with four congruent sides is a rhombus. **5.** 28° **7.** 10 **9.** 90° **11.** 14
13. Given: Parallelogram *WXYZ* is a rhombus
Prove: $\overline{WY} \perp \overline{XZ}$

Selected Answers

Topic 6

Proof:

Statements	Reasons
1) *WXYZ* is a rhombus	1) Given
2) $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$	2) Def. of rhombus
3) *X* and *Z* are equidistant from *W* and *Y*; *W* and *Y* are equidistant from *X* and *Z*	3) Def. of equidistant
4) *X* and *Z* are on the ⊥ bisector of $\overline{WY}$; *W* and *Y* are on the ⊥ bisector of $\overline{XZ}$	4) Converse of the Perpendicular Bisector Theorem
5) $\overline{XZ}$ is ⊥ bisector of $\overline{WY}$, $\overline{WY}$ is ⊥ bisector of $\overline{XZ}$	5) Two points determine a line.
6) $\overline{WY} \perp \overline{XZ}$	6) Def. of ⊥ bisector

15. 38.9 in.; Since $\triangle XYP \cong \triangle WZP$ and $\overline{PQ}$ is the altitude of $\triangle WXF$, $YZ = 2(PQ)$, so $YZ = 8$ and $WY = 14.4375$. Use the Pythagorean Theorem to find $XZ \approx 16.5$. **17.** Rectangle; let *x* be the side length of the square. The square has perimeter 4*x*. The length of the rectangle is equal to the length of the diagonal of the square, and the width of the rectangle is half the length of the diagonal of the square. The diagonal of the square is $\sqrt{2}x$, so the perimeter of the rectangle is $3\sqrt{2}x$. Since $3\sqrt{2} \approx 4.2$, the rectangle has a greater perimeter. **19.** 66° **21.** 8 **23.** 9.4 **25.** 76° **27.** 6.1 **29.** 5.7 **31.** 6 **33. a.** 25°; *DCAF* is a rhombus. Since $m\angle BAE = 90°$, $m\angle CAF = 50°$, so $m\angle CDF = 50°$. The diagonals of a rhombus bisect the angles, so $m\angle CDG = 25°$. **b.** 59.5 cm **35.** A, B, D **37. Part A** Zone 1: Each side is 4.9 ft. Zone 2: Each side is 4.4 ft. Zone 3: 3.3 ft by 10.5 ft **Part B** Zone 1: All angles are 90°. Zone 2: $m\angle F = m\angle H = 50°$, $m\angle E = m\angle G = 130°$; Zone 3: All angles are 90°. **Part C** Zone 1: 24.01 ft^2; Zone 2: 14.8 ft^2; Zone 3: 34.65 ft^2

Lesson 6-6

1. Knowing whether the diagonals of a parallelogram are perpendicular, congruent, or bisect opposite angles helps you classify the parallelogram. **3.** A parallelogram is a rectangle if and only if its diagonals are congruent. This statement is a combination of Theorem 6-18 and Theorem 6-21. **5.** rectangle **7.** rhombus **9.** 54 **11. Given:** *ABCD* is a parallelogram, $\overline{BD} \perp \overline{AC}$ **Prove:** *ABCD* is a rhombus. Since *ABCD* is a parallelogram, $\overline{BE} \cong \overline{DE}$ and $\overline{AE} \cong \overline{CE}$. Since $\overline{BD} \perp \overline{AC}$, $\angle AEB$, $\angle CEB$, $\angle CED$, and $\angle AED$ are all right angles and are therefore congruent. So by SAS, $\triangle AEB \cong \triangle CEB \cong \triangle CED \cong \triangle AED$. By CPCTC, $\overline{AB} \cong \overline{CB} \cong \overline{CD} \cong \overline{AD}$. Since *ABCD* is a parallelogram with four congruent sides, it is a rhombus. **13. Given:** *FGHJ* is a parallelogram, $\overline{GJ} \cong \overline{HF}$. **Prove:** *FGHJ* is a rectangle. Since *FGHJ* is a parallelogram, $\overline{GF} \cong \overline{HJ}$ and $\overline{GH} \cong \overline{FJ}$. It is given that $\overline{GJ} \cong \overline{HF}$. So, by SSS, $\triangle GJF \cong \triangle HFJ$, and $\angle GFJ \cong \angle HJF$ by CPCTC. Consecutive angles in a parallelogram are supplementary, and congruent

Selected Answers

Topic 6

supplementary angles are right angles. Thus, $\angle GFJ$ and $\angle HJF$ are right angles. Opposite angles in a parallelogram are congruent, so $\angle FGH$ and $\angle JHG$ are also right angles. *FGHJ* is a rectangle.
15. $y = -\frac{5}{4}x + 12$; For *WXYZ* to be a rhombus, the diagonals must be perpendicular. They also bisect each other because *WXYZ* must also be a parallelogram. First find the midpoint of $\overline{WY}$, which is (4, 7). Then find the slope of $\overline{WY}$, which is $\frac{4}{5}$. The slope of $\overline{XZ}$ must be the negative reciprocal, or $-\frac{5}{4}$. An equation of the line passing through (4, 7) with a slope of $-\frac{5}{4}$ is $y = -\frac{5}{4}x + 12$. **17.** Yes; the angle opposite the 100° angle must also be 100°, since they are opposite angles in a parallelogram. Using the sum of the angles in a triangle, the other two angles must be 40°. **19.** 68
21. parallelogram **23.** $x = 14$ **25.** Yes; the bottom left is a right triangle. The remaining portion is a square, because it is a parallelogram with congruent diagonals that are perpendicular. The area of the square is 12^2, and area of the right triangle is one quarter the area of the square. Multiply the area by 1.50 to get the price. **27.** 164 in. **29.** B

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** kite
5. midsegment of a trapezoid
7. 1,800°; 360° **9.** 59° **11.** No; the rod must be 42 in. because $\frac{1}{2}(12 + 42) = 27$.
13. 95° **15.** 12 **17.** If the length of each side along a vertical line is the same, then each quadrilateral is a parallelogram since vertical lines are parallel. **19.** 12 **21.** rectangle
23. 42; The diagram shows that *GHJK* is a parallelogram, so in order for *GHJK* to be a rhombus, the diagonals must be perpendicular. Thus, $48 + x = 90$, so $x = 42$.

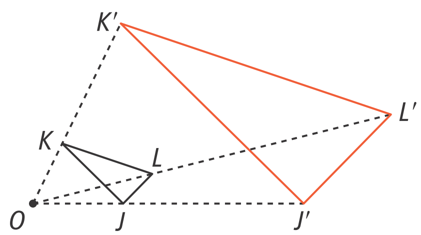
Selected Answers

Topic 7

Lesson 7-1

1. A dilation changes the side lengths of a figure in proportion to the scale factor of the dilation. The angles of the figure stay the same. **3.** All the rays go outward from the point that is the center of dilation.

5.



7. $P'(5, -15)$, $Q'(-15, -20)$, $R'(30, 10)$ **9.** $F'(0, -1)$, $G'(16, -1)$, $H'(16, -9)$, $J'(0, -9)$ **11.** The image points are $M'(ka, kb)$ and $N'(kc, kd)$. The slope of $\overleftrightarrow{MN}$ is $\frac{d-b}{c-a}$. The slope of $\overleftrightarrow{M'N'}$ is $\frac{kd-kb}{kc-ka} = \frac{k(d-b)}{k(c-a)} = \frac{d-b}{c-a}$. The slopes are equal, so the lines are parallel. **13.** perimeter: $\frac{2(a+b)}{k}$; area: $\frac{ab}{k^2}$ **15.** Yes; for any point (x, y), $D_{mn}(x, y) = (mnx, mny)$. But $D_m \circ D_n(x, y) = D_m(nx, ny) = (mnx, mny)$ **17.** The side lengths of the image are half the side lengths of the preimage. **19.** $A'(3, 0)$, $B'(12, -6)$, $C'(6, -9)$, $D'(-7.5, -15)$ **21.** 13 **23.** $(-1, 3)$ **25. a.** 8.4 m by 7 m. 7 m = 700 cm, and $\frac{700}{10} = 70$; Answers may vary. Sample: If he uses a scale factor of 70, he will use the entire height of the wall. The length will be 12 • 70 cm = 840 cm, or 8.4 m, which is less than the width available. **b.** 147,000 tiles; Across the width of the mosaic, he will need 700 ÷ 2 = 350 tiles, and along the height he will need 840 ÷ 2 = 420 tiles. 350 • 420 = 147,000 **27.** $(20, -8)$; $\frac{1}{3}$; $(-4, 0)$; 5 **29. Part A** Answers may vary. Sample: $\frac{1}{60}$; The wingspan of the glider is 9.8 m, or 980 cm, so the scale model would have a wingspan of $980 \bullet \frac{1}{60} \approx 16.3$ cm. **Part B** Sample: length: about 8.2 cm, wingspan: about 16.3 cm, height: 4 cm **Part C** Sample: about 78.6 cm^2; about 2.4 cm by 16.3 cm

Lesson 7-2

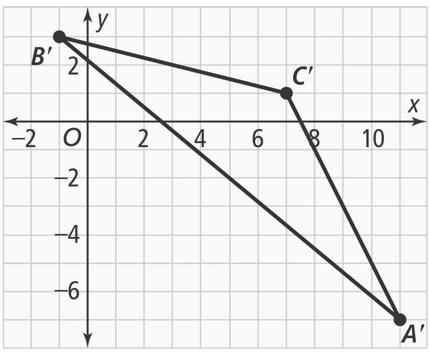
1. A similarity transformation includes a dilation and possibly one or more rigid motions. A similarity transformation produces a figure that has angles congruent to and sides proportional to the original figure. **3.** Both show the relationship between two figures as a result of transformations. Similarity transformations include dilation, and congruence transformations do not.
5. $A'(-0.5, 2.5)$, $B'(-2, -1.5)$, $C'(-1, 0)$, $D'(-3, 2)$ **7.** reflection, rotation, dilation **9.** yes; translate so that the centers align. Then dilate using the ratio of radii. **11.** $m\angle A' = 103°$, $m\angle B' = 100°$, $m\angle C' = 80°$, $m\angle D' = 77°$
13.
$$\begin{aligned}\left(D_k \circ T_{\langle a,b\rangle}\right)(x, y) &= D_k(x + a, y + b)\\ &= (kx + ka, ky + kb)\end{aligned}$$
$$\begin{aligned}\left(T_{\langle a,b\rangle} \circ D_k\right)(x, y) &= T\langle a, b\rangle(kx, ky)\\ &= (kx + a, ky + b)\end{aligned}$$
So order is important for translation and dilation.
15. $X'(14, -7)$, $Y'(11, -2.5)$, $Z'(2, 0.5)$
17. $P'(-6, -2)$, $Q'(-4, 8)$, $R'(-10, 0)$
19. Answers may vary. Sample: down

Selected Answers

Topic 7

2 units and right 4 units, then dilation with P' as center and scale factor 2; $PQRS \sim DABC$ **21.** Answers may vary. Sample: rotation and dilation; $HJKL \sim ABCD$ **23.** No; the bases of the trapezoids are congruent but the top trapezoid appears to be shorter in height than the bottom trapezoid. **25.** 27.5 m^2; Answers may vary. Sample: 5.5 m is represented by 11 sections on the grid, so each grid line represents 0.5 m. The scrim is 10 grid sections tall, or 5 m. 5 m × 5.5 m = 27.5 m^2

27.



29. Part A Check students' work.
Part B Check students' work.
Part C Yes; all figures are similar to each other, so there is a similarity transformation that maps any figure to any other figure.

Lesson 7-3

1. A similarity transformation is a composition of a dilation and a rigid motion. Dilation and rigid motion preserve angle measures, so similar triangles have congruent angles. Dilation makes side lengths proportional and rigid motion preserves side lengths, so corresponding sides of similar triangles are proportional.

3. No; by the Triangle Angle-Sum Theorem, $m\angle D = 40$ and $m\angle T = 55$. So $\angle D \cong \angle R$, $\angle E \cong \angle S$, and $\angle F \cong \angle T$. By the AA~ Theorem, $\triangle DEF \sim \triangle RST$.
5. All corresponding sides are proportional, which means $\frac{3.6}{9} = \frac{2.84}{7.1} = \frac{2.24}{5.6} = 0.4$, so $\triangle GHJ \sim \triangle KLM$ by SSS~.
7. $a = 94$. **9.** 6 m

11. a. $\triangle NLM$; $\frac{AB}{NL} = \frac{BC}{LM} = \frac{AC}{NM}$, so $\triangle ABC \sim \triangle NLM$ by SSS~.

b. $\triangle TSR$; $\angle A \cong \angle T$, and $\angle B \cong \angle S$, so $\triangle ABC \sim \triangle TSR$ by AA~.

13. Russell assumed that either $\angle K$ or $\angle M$ has a measure of 60°, which is only true if the triangles are similar, which has not yet been proven. $\angle K$ could measure 59° and $\angle M$ could measure 41°, and the triangles would not be similar. **15.** Because there is no Side-Side-Angle congruence criterion, it cannot be shown that a dilated triangle is necessarily congruent to a triangle where the two pairs of corresponding sides are proportional and a nonincluded pair of angles is congruent. **17.** Yes; $\angle K = 180° - 60° - 40° = 80°$, so $\triangle JKL \sim \triangle PQR$ by AA~.
19. 2.8 **21.** Dilate $\triangle ABC$ by a factor of $\frac{EF}{AB}$. The image $\triangle A'B'C'$ is congruent to $\triangle EFG$ by SSS, so there is a rigid motion that maps $\triangle A'B'C'$ to $\triangle EFG$. Since the composition of a dilation and rigid motion maps $\triangle ABC$ to $\triangle EFG$, $\triangle ABC \sim \triangle EFG$. **23.** about 77 meters; The triangles on either side of the lens are similar because the angles are vertical angles and each pair of black dashed lines is congruent. Two pairs of congruent sides must be similar to

Selected Answers

Topic 7

each other. Consider the right triangles that have the black dashed lines as hypotenuses. $\frac{d}{0.1} = \frac{1}{0.0013}$, so $d \approx 77$. **25.** A, B **27. Part A** 12 km; The right triangle between the mountain and the camp is similar to the right triangle between the camp and the helicopter by AA ~, because they share the angle from the camp. $\frac{d}{3.5} = \frac{d - 2.4}{2.8}$, so $d = 12$ km. **Part B** 7.5 minutes; $\frac{12}{1.6} = 7.5$ **Part C** 4.8 km; A distance of 5 km along the black dashed line from the campsite creates another similar triangle. The horizontal leg has a length of 4.8 km.

Lesson 7-4

1. The altitude to the hypotenuse divides a right triangle into two triangles that are similar to each other and to the original triangle. The altitude divides the hypotenuse into two segments, and the length of the altitude is the geometric mean of the lengths of the segments. The length of the short leg of the triangle is the geometric mean of the length of the hypotenuse and the length of the segment that is adjacent to the short leg. The length of the long leg of the triangle is the geometric mean of the length of the hypotenuse and the length of the segment that is adjacent to the long leg. **3.** The arithmetic mean and geometric mean of two numbers are between the values of the two numbers. The arithmetic mean is found by adding the two numbers and multiplying the sum by one-half. The geometric mean is found by multiplying the two numbers and then raising the product to the one-half power. **5.** $9\sqrt{10}$ **7.** 4 **9.** $4\sqrt{5}$ **11. a.** Dilate $\triangle XYZ$ by the scale factor of $\frac{XZ}{XY}$ with center of dilation at Z. Reflect the image across the perpendicular bisector of $\overline{Z'Y'}$. Then rotate about point Z by $m\angle WZY$. **b.** Dilate $\triangle XYZ$ by the scale factor of $\frac{XZ}{XY}$ with center of dilation at Z. Reflect the image across the perpendicular bisector of $\overline{Z'Y'}$. Then rotate about point Z by $m\angle WZY$. **13.** No; $\overline{AD}$ is not the altitude to the hypotenuse, so AD is not the geometric mean of BD and CD. **15.** An isosceles right triangle; consider right triangle ABC with hypotenuse $\overline{BC}$ and $\overline{AD}$ the altitude to $\overline{BC}$. Then by Corollary 1, $\frac{BD}{AD} = \frac{AD}{CD}$. Since $\overline{AD}$ bisects $\overline{BC}$, $BD = CD$, so $BD^2 = AD^2$, $BD = AD$, and $\triangle ABD$ is an isosceles right triangle. Since $\triangle ABC \sim \triangle ABD$, $\triangle ABC$ is also an isosceles right triangle. **17.** 60 and 25; $\frac{h}{144} = \frac{65}{156}$, so $h = \frac{(65)(144)}{156} = 60$, and $\frac{x}{60} = \frac{65}{156}$, so $x = \frac{(65)(60)}{156} = 25$. **19.** $a = 16$ and $b = 49$; By Corollary 2 to Theorem 7-4, $\frac{a}{20} = \frac{20}{25}$, so $25a = (20)(20) = 400$, and $a = 16$. By Corollary 2 to Theorem 7-4, $\frac{b}{175} = \frac{175}{625}$, so $625b = (175)(175) = 30{,}625$, and $b = 49$. **21.** 8; All right triangles in the figure are similar by Theorem 7-4. The leg labeled w in the triangle with hypotenuse length 10 corresponds to the leg labeled 4 in the 3-4-5 triangle. $\frac{w}{10} = \frac{4}{5}$, so $w = 8$. **23. a.** 144.5 cm and 120 cm; $x = \sqrt{68^2 + 127.5^2} = 144.5$; $\frac{y}{68} = \frac{127.5}{144.5}$, so $y = 60$

Selected Answers

Topic 7

b. At the center of the shorter crossbar (60 cm) and 32 cm (or 112.5 cm) from one end of the longer crossbar; Use Corollary 2 to Theorem 7-4 to find the location on the larger crossbar: $\frac{d}{68} = \frac{68}{144.5}$, so $d = 32$, which is 112.5 cm from the other end of the crossbar.)
25. Answers may vary. Sample: $\frac{7}{x} = \frac{x}{9}$

Answers may vary. Sample: $\frac{x}{18} = \frac{18}{21}$

27. Part A 5.51 m **Part B** 2.54 m
Part C 3.3 m; Let x be the distance from the tree and y be the height of the tree. A proportion from the triangle formed by Tia is $\frac{1.6}{x} = \frac{x}{y - 1.6}$, and a proportion from the triangle formed by Felix is $\frac{1.7}{x} = \frac{x}{y - 1.7}$. So, $x^2 = 1.6(y - 1.6)$, and $x^2 = 1.7(y - 1.7)$. Solve the equation $1.6\,(y - 1.6) = 1.7(y - 1.7)$ to get $y = 3.3$.

Lesson 7-5

1. The segments of each transversal are divided proportionally, so segments of the sides of a triangle are divided proportionally. **3.** The additional information needed is that the segment of length x bisects both sides of the triangle. **5.** By the Triangle-Angle-Bisector Theorem, $\frac{LP}{LM} = \frac{NP}{NM}$. Assume $LP > LM$. Then $NP > NM$, and $LP + NP > LM + NM$. This contradicts the Triangle Inequality Theorem, so $LP < LM$ **7.** 14
9. Current line segment lengths lead to the following proportion:

$$\frac{2x + 1}{12} = \frac{x - 1}{8}$$
$$\frac{2x + 1}{3} = \frac{x - 1}{2}$$
$$4x + 2 = 3x - 3$$
$$x = -5$$

The value of x being negative 5 would lead to one of the segments to have length negative 6 which is impossible. To solve this issue and to have the answer $x = 5$, change the segment length from $2x + 1$ to $x + 1$. **11.** 3
13. 60%; By the Triangle-Angle-Bisector Theorem, $\frac{8}{6} = \frac{12}{SR}$, so $SR = 9$. The area of $\triangle PQR$ is $15h$, and the area of $\triangle QRS = 9h$, where h is the height from vertex Q, so the percent area is $\frac{9h}{15h} \times 100\% = 60\%$.
15. By the Triangle Midsegment Theorem and by definition of midpoint, $OP = \frac{1}{2}LN = LQ = QN$, $OQ = \frac{1}{2}MN = MP = PN$, and $PQ = \frac{1}{2}ML = MO = OL$. By SSS, $\triangle LOQ \cong \triangle OMP \cong \triangle QPN \cong \triangle PQO$.
17. 6.5 **19.** $\frac{13}{3}$ **21.** 7.5 **23.** 8.4
25. Draw $\overline{AF}$ and let G be the point of intersection of $\overline{AF}$ and $\overline{BE}$. Apply the Side-Splitter Theorem to $\triangle ACF$, so $\frac{CB}{BA} = \frac{FG}{GA}$, or $\frac{AB}{BC} = \frac{AG}{GF}$. Apply the Side-Splitter Theorem to $\triangle FDA$, so $\frac{DE}{EF} = \frac{AG}{GF}$. By the Transitive Property of Equality, $\frac{AB}{BC} = \frac{DE}{EF}$. **27.** 25 ft **29.** 8
31. Part A $w = 6$; $x = 5.625$; $y = 1.25\,z$
Part B 12, 9, 6.75, 5.0625, $y = 1.125\,z$
Part C Sample: the line that has length 6.75, because that length is closest to one-half of the line with length 12

Selected Answers

Topic 7

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** similarity transformation **5.** scale factor
7. $K'(0, 4)$, $L'(9, -8)$, $M'(-6, 4)$
9. No; the scale factor is the ratio of a side length of $\triangle PQR$ to the corresponding side length of $\triangle ABC$, so $\frac{PQ}{AB} = \frac{5}{10} = \frac{1}{2}$. **11.** $\frac{KL}{NL} = \frac{2}{4} = \frac{1}{2}$, $\frac{LN}{LM} = \frac{4}{8} = \frac{1}{2}$, and $\frac{KN}{NM} = \frac{3}{6} = \frac{1}{2}$, so $\frac{KL}{NL} = \frac{LN}{LM} = \frac{KN}{NM}$. Thus, $\triangle KLN \sim \triangle NLM$ by SSS $\sim$.
13. 8 **15.** $4\sqrt{3}$ **17.** 14.4 **19.** 3

Selected Answers

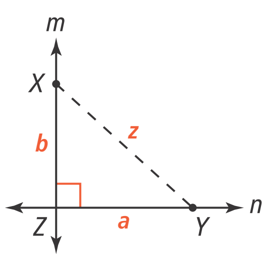
Topic 8

Lesson 8-1

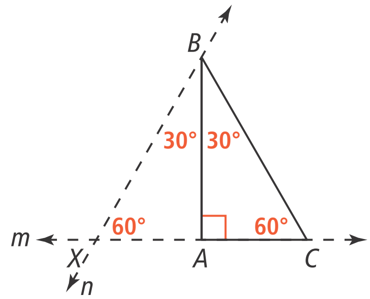
1. If you draw the altitude to the hypotenuse in any right triangle, you divide the triangle into two similar triangles that are also similar to the original triangle. By setting up proportions between the side lengths of the three triangles, you can prove the Pythagorean Theorem. **3.** Let the length of the short leg be s. Since $s =$ 4.5, the length of the long leg, $(4.5)\sqrt{3}$, is $s\sqrt{3}$, and the length of the hypotenuse, 9, is $2s$. These are the relationships between the lengths of the legs and hypotenuse of a 30°-60°-90° triangle. **5.** $x = 9\sqrt{3}$
7. No; $35^2 + 36^2 \stackrel{?}{=} 71^2$

$$2{,}521 \neq 5{,}041$$

9. No; the pennant is a 30°-60°-90° triangle, so the length of the hypotenuse is 2 times the length of the short leg. The short leg is the widest part of the pennant. It is $41 \div 2 = 20.5$ inches. The pennant is too wide for the 19-inch space between the windows.
11. Triangle DEF is a 45°-45°-90° triangle. So, $14 = \sqrt{2}(EF)$, or $EF = 7\sqrt{2}$.
13. If $(XZ)^2 + (XY)^2 > (YZ)^2$, then $YZ < \sqrt{(XY)^2 + (XZ)^2}$ and $m\angle YXZ < 90°$. Therefore, $\triangle XYZ$ would be an acute triangle. By the same reasoning, if $(XZ)^2 + (XY)^2 < (YZ)^2$, $m\angle YXZ > 90°$ and $\triangle XYZ$ would be an obtuse triangle.
15. 13 **17.** Given: $\triangle ABC$, where $a^2 + b^2 = c^2$
Prove: $\triangle ABC$ is a right triangle.



Draw perpendicular lines m and n intersecting at Z. On line m plot X so that $XZ = b$. On line n plot Y so that $YZ = b$. Draw $\overline{XY}$ with length z. By the Pythagorean Thm. $a^2 + b^2 = z^2$. Given that $a^2 + b^2 = c^2$ and by the Transitive Prop of $= c^2 = z^2$, so $c = z$. By SSS, $\triangle XYZ \cong \triangle ABC$, and $\angle X \cong \angle A$ by CPCTC. Therefore, $m\angle A =$ 90° and $\triangle ABC$ is a right triangle.
19. Given: $\triangle ABC$
Prove: 1) $BC = 2AC$ and 2) $AB = AC\sqrt{3}$



Draw line m through $\overline{AC}$. Draw line n through B and intersecting line m at a 60° angle at point X to form equilateral $\triangle XBC$. 1) $\overline{AB}$ is an altitude of $\triangle XBC$ and bisects $\overline{XC}$. So, $XC = 2AC$. Because $\triangle XBC$ is equilateral, $XC = BC$, and $BC =$ $2AC$. 2) By the Pythagorean Theorem,

$$(AC)^2 + (AB)^2 = (BC)^2.$$
$$(AC)^2 + (AB)^2 = (2AC)^2$$
$$(AB)^2 = 3(AC)^2$$
$$AB = AC\sqrt{3}$$

21. $XY = 9$, $YZ = 9\sqrt{3}$ **23.** Between 4.33 and 4.62 in.; Answers may vary. Sample: The height is the long leg of a 30°-60°-90° triangle, so the height is $\sqrt{3}d$. $7.5 < \sqrt{3}d < 8$, so $4.33 < d < 4.62$. **25.** Yes; the height of the chocks are $\sqrt{9^2 - 6^2} \approx$ 6.7 in. One fourth the height of the wheels is 6 in. **27.** A

Selected Answers

Topic 8

Lesson 8-2

1. For all right triangles with a given acute angle, the ratios of corresponding side lengths are constant because all such triangles are similar. **3.** Using an inverse trigonometric function is exactly like using inverse operations. If the cosine of an angle is calculated, taking the arccosine of the result returns the original angle, just as if you had added four and then subtracted four from the angle. **5.** Use the Pythagorean Theorem to find the length b of the other leg and then $\cos A = \frac{b}{c}$. **7.** $\frac{3}{5}$ **9.** $\frac{5\sqrt{34}}{34}$ **11.** approx. 30.96° **13.** sine: $\frac{3}{5}$; cosine: $\frac{4}{5}$ **15.** $m\angle S \approx 23.58°$, $m\angle T \approx 66.42°$ **17.** $3x\sqrt{3}$ **19. a.** $\frac{\sqrt{2}}{2}$ **b.** 45° **21.** $\frac{a}{c}$ **23.** $\frac{a}{b}$ **25.** $\frac{4\sqrt{17}}{17}$ **27.** $\frac{4\sqrt{17}}{17}$ **29.** 4 **31.** $\frac{1}{2}$ **33.** 1 **35.** $\sqrt{3}$ **37.** ≈10.50 **39.** ≈53.13° **41.** ≈71.57° **43.** ≈56.25° **45.** Approximately 36.87°; no matter what amount of time (in seconds) t it takes to reach the receiver, the the ball will travel $20t$ ft and the receiver will run $12t$ ft. The sine of x is therefore 0.6, and the measure of the angle is $\sin^{-1}(0.6) \approx 36.87°$. **47. I.** C **II.** D **III.** A **IV.** B **49. a.** yes; 1.0 ft **b.** 10.8 ft

Lesson 8-3

1. The sides and angles of any triangle are related by the Law of Sines. The Law of Sines states that the sine of an angle of a triangle divided by the length of the opposite side is equal for all vertices of the triangle. Thus, you can set up and solve a proportion to determine a missing side length or angle measure using the Law of Sines. **3.** $\angle L$, MN; $\angle M$, LN; $\angle N$, LM; The Law of Sines can be used to find LM using the given information. By using the Triangle Angle-Sum Theorem to find $m\angle M$, LN can also be found. **5.** EF, ED **7.** $m\angle Q = 38.1°$, $m\angle R = 43.9°$ **9.** 13.2 **11.** 30.0 **13.** Kimberly did not find the inverse sine of the value she calculated. **15.** The third angle can be found by subtracting the other two angles from 180°, and then either or both of the other sides can be found using the Law of Sines. **17.** Use the Law of Sines to get $QR = \frac{9 \sin 42°}{\sin 37°}$. Use the Triangle Angle-Sum Theorem to find that $m\angle Q = 101°$. Then use the Law of Sines to get $PR = \frac{9 \sin 101°}{\sin 37°}$. An expression for the perimeter is $9 + \frac{9 \sin 42°}{\sin 37°} + \frac{9 \sin 101°}{\sin 37°}$. **19.** 43.7 **21.** 17.3 **23.** 15.8 **25.** 18.3° **27.** 36.2° **29.** 74.6° **31.** 50.1 **33.** 115.4 **35.** 6,533 ft **37.** 4.6 **39. Part A** 17.9 ft **Part B** She does not have enough information to find the length of either beam. She can calculate the lengths of $\overline{AB}$ and $\overline{CD}$, and the angles those beams make with the walkway. Thus she needs to know any other angle or length in the triangles formed by the new beams. To find the lengths, she should measure the walkway from the left end to point B and from the right end to point C, or measure the angles the new beams make with the walkway, or measure the angles the new beams make with the bottom ends of $\overline{AB}$ and $\overline{CD}$, or measure the distance from the ends of the walkway to the bottom ends of $\overline{AB}$ and $\overline{CD}$.

Selected Answers

Topic 8

Lesson 8-4

1. The sides and angles of any triangle are related by the Law of Cosines. The Law of Cosines allows the calculation of any angle of a triangle if three sides are known, or the third side when two sides and the included angle are known. **3.** The square of a side of a triangle is equal to the sum of the squares of the other two sides minus twice the product of the two sides and the cosine of their included angle.
5. Let $AB = c$, $BC = a$, and $AC = b$;
By the Law of Cosines,
$c^2 = a^2 + b^2 - 2ab\cos 90^\circ$
$(c^2 - a^2 - b^2) \div (-2ab) = \cos 90^\circ$
However, by the Pythagorean Theorem,
$a^2 + b^2 = c^2$
$c^2 - a^2 - b^2 = 0$
By substitution,
$0 \div (-2ab) = \cos 90^\circ$
$0 = \cos 90^\circ$
7. p **9.** $m\angle R$, $m\angle S$, $m\angle T$ **11.** 52.6
13. You set up an equation with the missing angle measure and then solve the equation for the cosine of the missing angle. Then you take the $\cos^{-1}$ of the result to get the angle measure.
15.

$$\left(\sqrt{4a^2 + b^2}\right)^2 = a^2 + c^2 - 2ac\cos(180^\circ - x^\circ)$$

$$\frac{4a^2 + b^2 - a^2 - c^2}{-2ac} = \cos(180^\circ - x^\circ)$$

$$\frac{3a^2 + (b^2 - c^2)}{-2ac} = \cos(180^\circ - x^\circ)$$

$$\frac{3a^2 - a^2}{-2ac} = \cos(180^\circ - x^\circ)$$

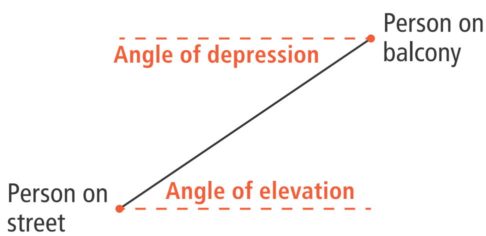
$$-\frac{a}{c} = \cos(180^\circ - x^\circ)$$

17. a. Set up the equation $a^2 = b^2 + c^2 - 2bc\cos A$, and solve for $\cos A$. Then find the $\cos^{-1}$ of the result.
b. Set up the equation $a^2 = b^2 + c^2 - 2bc\cos A$, and solve for a^2. Then find the square root of the result.
19. $f = \sqrt{d^2 + e^2 - 2de\cos F}$ **21.** 5.9
23. 4.4 **25.** 72.6 **27.** 78.6
29. Yes; let the distance from Damian to Camilla be a. By the Law of Cosines:
$a = \sqrt{7^2 + 10^2 - 2(7)(10)\cos 65}$
$a \approx 9.4781$
$9.4781 < 9.5$, so Camilla is within Damian's range of accuracy. **31.** 2.77 ft; 51.6° **33.** D

Lesson 8-5

1. Trigonometric ratios, the Law of Sines, and the Law of Cosines are used to find lengths and angles of triangles.

3. Sample:



5. As the elevator rises, the angle of depression increases. **7.** 83.6 units^2
9. 992.2 ft^2 **11.** Leah set up the formula for the Law of Cosines, but used the sine function. **13. a.** $x = \frac{b}{\tan y^\circ}$
b. $x = \frac{a\sin y^\circ}{\sin z^\circ}$ **15.** 16.7° **17.** 13 ft 11.6 in.
19. 93.7 units^2 **21.** 282.1 units^2
23. 29.2 min **25.** 54.2 ft **27.** D

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** Law of Sines
5. Pythagorean triple **7.** $\frac{\sqrt{39}}{8}$ **9.** Yes; the two ropes form the hypotenuses of right triangles with height of 5 feet.

Selected Answers

Topic 8

The length of the two ropes is $2\left(\frac{5}{\sin 40^\circ}\right)$, or about 15.6 feet, and $15.6 < 16$.
11. 35.3° **13.** 27.9 **15.** 28 in.
17. 66.3 m^2 **19.** 95.3 ft^2 **21.** Model the situation as a right triangle with height 58 + 5.5, or 63.5 feet. The angle of depression and the angle formed by the river and the line of sight are alternate interior angles, so they are congruent. Let w be the width of the river. Then $\tan 74^\circ = \frac{63.5}{w}$ or $w \approx 18.2$. The river is about 18.2 feet wide.

Selected Answers

Topic 9

Lesson 9-1

1. The properties of geometric figures plotted on the coordinate plane, such as length, parallel sides, existence of right angles, or location of midpoints, can be described using the formulas for distance, slope, and midpoint.
3. Samples: Use the Distance Formula to show that opposite sides are congruent. Use the Slope Formula to show that opposite sides are parallel. Use the Midpoint Formula to show that the diagonals bisect each other.
5. They bisect each other. Midpoint of $\overline{JL} = (4.5, 5)$; midpoint of $\overline{KM} = (4.5, 5)$. Since diagonals $\overline{JL}$ and $\overline{KM}$ have the same midpoint, they bisect each other.
7. Isosceles right triangle; $PQ = \sqrt{50}$, $QR = 5$, $PR = 5$; slope of $\overline{QR} = -\frac{3}{4}$, slope of $\overline{PR} = \frac{4}{3}$. Since $QR = PR$ and product of the slopes is -1, $\triangle PQR$ is an isosceles right triangle. **9.** Kelley assumed that $\triangle PQR$ is a right triangle.
11. By the distance formula, $AB = DE = \sqrt{10}$, $BC = EF = \sqrt{26}$, and $AC = DF = \sqrt{20}$. So, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$. By SSS, $\triangle ABC \cong \triangle DEF$. **13.** Use the slope and one point on p to write an equation for p in point-slope form. Then rewrite the equation in slope-intercept form. The slope m of p is the negative reciprocal of the slope of the line through $\overline{AB}$. So, $m = -\left(\frac{x_2 - x_1}{y_2 - y_1}\right)$. Since p bisects $\overline{AB}$, the midpoint $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ of $\overline{AB}$ is on p. So, in point slope form, the general equation for line p is

$$y - \left(\frac{y_1 + y_2}{2}\right) = -\left(\frac{x_2 - x_1}{y_2 - y_1}\right)\left[x - \left(\frac{x_1 + x_2}{2}\right)\right]$$

In slope intercept form,

$$y = -\left(\frac{x_2 - x_1}{y_2 - y_1}\right)x + \frac{x_1^2 - x_2^2 + y_1^2 - y_2^2}{2(y_1 - y_2)}$$

15. $m \perp p$ **17.** $\left(\frac{3}{2}, 0\right)$ **19.** Isosceles triangle, but not a right triangle; it is isosceles because $AD = CD = 5$ units and $AC = 2\sqrt{5}$ units. It is not right triangle because $\overline{AD}$ is a vertical line, the slope of the line through $\overline{AC}$ is -3, and the slope of the line through $\overline{CD}$ is $\frac{3}{4}$, so no sides are perpendicular to each other.
21. Kite; it has two pairs of congruent, consecutive sides. $AB = BC = \sqrt{10}$ units and $CD = AD = 5$ units. So, $\overline{AB} \cong \overline{BC}$ and $\overline{CD} \cong \overline{AD}$. **23.** area = 15 square units; perimeter = $10 + 2\sqrt{10}$ units
25. (5, 1); parallelogram **27.** Set up axes for a coordinate plane so the dashed line goes through the origin and is represented by the equation $y = x$. Then assign coordinates to each vertex of the drawing. Show that there are pairs of vertices that map to each other by the reflection across the line $y = x$.

29.

	Yes	No
$\overline{AB} \cong \overline{AC}$	☑	☐
$BC = AB\sqrt{2}$	☑	☐
The midpoint of BC is (5.5, 4).	☐	☑
The perimeter is 12.5 units.	☐	☑

31. Part A Dana is correct; $\overline{AD} \parallel \overline{BC}$ and $AB = CD = 2\sqrt{5}$, so $ABCD$ is an isosceles trapezoid. **Part B** 3 ft
Part C legs: $\frac{\sqrt{29} - 2\sqrt{5}}{2}$ ft longer; support: same length, 3 ft

Selected Answers

Topic 9

Lesson 9-2

1. In a coordinate proof, relationships of lines and angles are represented algebraically, using the coordinates of a figure drawn on a coordinate plane.
3. A coordinate geometry proof is a proof that uses algebraic relationships to prove a geometric theorem or solve a geometric problem. **5.** $A(-a, 0)$, $B(0, b)$, $C(a, 0)$; place the altitude to the vertex on the y-axis to make calculations of the lengths of the segments of the base convenient. **7.** Draw the triangle on the coordinate plane. Find AC.
Find the length of the altitude from vertex B. Apply the triangle area formula. **9.** Draw $\overline{AB}$ with $A(0, 0)$ and $B(2a, 0)$. By the midpoint formula, point P, the midpoint of $\overline{AB}$, has coordinates $P(a, 0)$. By the Distance Formula, $AP = BP = a$. **11.** Draw two squares—one with vertices (0, 0), $(a, 0)$, (a, a), and $(0, a)$, and the other with vertices (0, 0), $(b, 0)$, (b, b), and $(0, b)$. These represent any two squares that can be mapped by rigid motions so a vertex is placed at the origin and two sides are along the two positive axes. Then a dilation centered at the origin with scale factor $\frac{b}{a}$ maps the first square to the second square. Because a similarity transformation exists, the two squares are similar. **13.** $\left(-\frac{a}{2}, 0\right)$, $\left(\frac{a}{2}, 0\right)$, $\left(0, \frac{a\sqrt{3}}{2}\right)$; these coordinates give a triangle with side lengths a. **15.** Find the slope of each side. Show that the product of the slopes of two sides is -1. **17.** $y = -x + a$
19. Draw a circle centered on the origin, with point $A(a, 0)$ where the circle intersects the positive x-axis and point $B(-a, 0)$ where the circle intersects the negative x-axis. Find OA and AB. Show that $AB = 2(OA)$.
21. Draw $\triangle XYZ$ with vertices $X(2a, 2b)$, $Y(0, 2c)$, and $Z(0, 0)$ so that side $\overline{YZ}$ is on the x-axis. Then find the coordinates of W and show that area of $\triangle XYZ$ equals 2(area of $\triangle XWZ$). **23.** Draw a right triangle with vertex A at the origin, vertex $B(0, a\sqrt{3})$, and vertex $C(0, a)$. Draw a second right triangle with vertex A at the origin, vertex $D(0, b\sqrt{3})$, and vertex $E(0, b)$. These represent any two triangles with an acute angle measuring 30° that can be mapped by rigid motions so the vertex of the right angle is placed on the origin and the two adjacent sides are along the two positive axes. Then a dilation centered at the origin with scale factor $\frac{b}{a}$ maps one triangle to the other. Because a similarity transformation exists, the two triangles are similar. **25.** Draw $\triangle ABC$ with $A(0, 0)$, $B(0, 2a)$, and $C(2b, 0)$. Using the Midpoint Formula, the midpoint of $\overline{BC}$ is $P(a, b)$. Using the Distance Formula, the length BC of the hypotenuse is $2\sqrt{a^2 + b^2}$ and the length AP of the median is $\sqrt{a^2 + b^2}$. **27.** The point two-thirds of the way along the way from point C to the midpoint $(b, 0)$ of $\overline{AC}$ is $\left(b, \frac{b\sqrt{3}}{3}\right)$ and is the centroid from the Concurrency of Medians Theorem. The equation of the line that is the perpendicular bisector of $\overline{AC}$ is $y = -\frac{1}{\sqrt{3}}(x - 2b)$ and the equation of the line that is the perpendicular bisector of $\overline{AB}$ is $x = b$. The point of intersection is $\left(b, \frac{b}{\sqrt{3}}\right)$, or $\left(b, \frac{b\sqrt{3}}{3}\right)$.

Selected Answers

Topic 9

This is the circumcenter from the Concurrency of Perpendicular Bisectors Theorem. The centroid and circumcenter of an equilateral triangle are the same point. **29.** Let $A(-2a, 0)$, $B(-2b, 2c)$, $C(2b, 2c)$, and $D(2a, 0)$. The midpoints of the sides are $E(-a-b, c)$, $F(0, 2c)$, $G(a+b, c)$, and $H(0, 0)$. Then $EF = FG = GH = EH = \sqrt{(a+b)^2 + c^2}$. Since all four sides are congruent, $EFGH$ is a rhombus. **31.** labeled diagram, algebra, coordinates

33. Part A $D\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$;

$E\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$; $F\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Part B By using the formula for partitioning, the coordinates of point P, $\frac{2}{3}$ of the way from A to D, are

$$(x_1, y_1) + \frac{2}{3}\left[\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right) - (x_1, y_1)\right]$$

$$= (x_1, y_1) + \left(\frac{x_2 + x_3}{3}, \frac{y_2 + y_3}{3}\right) - \left(\frac{2x_1}{3}, \frac{2y_1}{3}\right)$$

$$= \left(\frac{x_1}{3}, \frac{y_1}{3}\right) + \left(\frac{x_2 + x_3}{3}, \frac{y_2 + y_3}{3}\right)$$

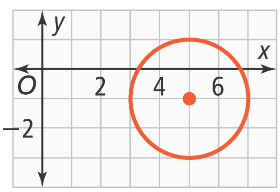
$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Part C It does not make sense because x_1 and y_1 would both be zero and the solution would not generalize that the coordinates of the centroid is the average of all three coordinates.

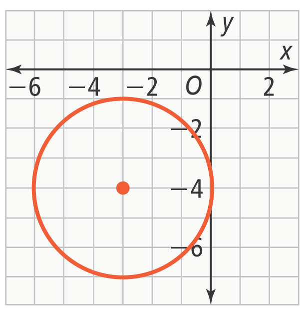
Lesson 9-3

1. The radius is the length of the segment connecting any point on the circle and the center. The equation is determined from the radius and the coordinates of the center. **3.** Yes; use the Distance Formula to find the distance between the center and point. This distance is the radius of the circle. Then substitute the values for the radius and coordinates of the center into the equation. **5.** center: (4, 9); radius: 1 **7.** center: (–7, 1); radius: 3 **9.** $x^2 + y^2 = 5$ **11.** yes **13.** Given: $\odot Q$ with center (h, k) and radius r Prove: An equation of $\odot Q$ is $(x-h)^2 + (y-k)^2 = r^2$. Proof: Let $P(x, y)$ be any point on $\odot Q$. By def. of circle and radius, the distance from P to Q is r. By the Distance Formula, $D(Q, P) = \sqrt{(x-h)^2 + (x-k)^2}$, so $\sqrt{(x-h)^2 + (y-k)^2} = r$. Square each side to get $(x-h)^2 + (y-k)^2 = r^2$. **15.** The equation should be $x^2 + (y+2)^2 = 49$, so the point is not on the circle. **17.** $(x-2)^2 + (y-4)^2 = 25$ **19.** Both students are correct. The graph shows a circle of radius 3 with center (–2, 2). The center of any circle with radius 3 that passes through (–2, 2) must be 3 units from (–2, 2), which points on the circle. **21.** center: (–2, 0), radius: $\sqrt{13}$ **23.** center: (8, 12), radius: $4\sqrt{6}$ **25.** $(x-1)^2 + y^2 = 2$ **27.** $(x+3)^2 + (y-8)^2 = 81$ **29.** $(x+5)^2 + (y+9)^2 = 13$ **31.** yes

33.



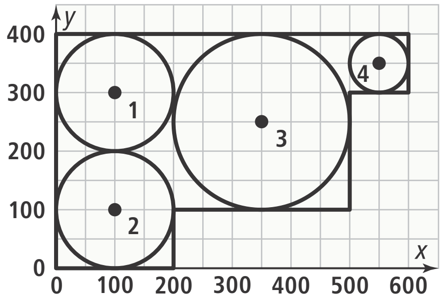
35.



Selected Answers

Topic 9

37. inside; The distance between (7, 2) and (4, 0) is $\sqrt{13}$, which is less than 5.
39. yes; (100, 100) **41.** B, C, D, E, F
43. Part A Sample:



Sprinkler 1: center: (100, 300), radius: 100, equation:
$(x - 100)^2 + (y - 300)^2 = 100^2$

Sprinkler 2: center: (100, 100), radius: 100, equation:
$(x - 100)^2 + (y - 100)^2 = 100^2$

Sprinkler 3: center: (350, 250), radius: 150, equation:
$(x - 350)^2 + (y - 250)^2 = 150^2$

Sprinkler 4: center: (550, 350), radius: 50, equation: $(x - 550)^2 + (y - 350)^2 = 50^2$

Part B Sample (based on Part A Sample):
total area of field: 180,000 m^2
total area irrigated: $\approx$141,300 m^2
percent of field irrigated: 78.5%

Lesson 9-4

1. A parabola is defined in terms of a focus and directrix. The equation uses p, which is half the distance between the focus and directrix, and the vertex, which is the midpoint of the segment connecting the focus and directrix.
3. Sample: The word *direction* makes me think of something moving in a direction, and the points in a line continue on forever in two directions. So, the directrix is a line. **5.** The value of p is the distance between the focus and vertex, which is $b - c$. So, write the equation of a parabola, using $h = a$, $k = b$, and $p = b - c$.
7. $y + 2 = \frac{1}{12}(x - 5)^2$ **9.** $y + \frac{5}{2} = \frac{1}{6}(x - 2)^2$
11. vertex: (0, 2), focus: $\left(0, \frac{7}{2}\right)$, directrix: $y = \frac{1}{2}$ **13.** vertex: (9, –3), focus: (9, 2), directrix: $y = -8$
15. $y - 7 = \frac{1}{4}(x + 1)^2$
17. $y + 4 = \frac{1}{12}(x + 3)^2$
19. a. The focus must be as far above the vertex as the directrix is below the vertex. The focus is (–1, 5). **b.** The value of p is 1 and the vertex is (–1, 4), so an equation is $y - 4 = \frac{1}{4}(x + 1)^2$.
c. The vertex can be read from the graph. The vertex is (–1, 4). **21.** focus: $\left(3, \frac{1}{4}\right)$, vertex: (3, 0), directrix: $y = -\frac{1}{4}$.
23. The value of p is $\frac{1}{4}$ and the vertex is (0, 0), so an equation is $y = x^2$.
25. (6, –1) **27.** $\left(-2, -\frac{7}{2}\right)$ **29.** vertex: (1, –4), focus: (1, 5), directrix: $y = -13$
31. vertex: (10, –5), focus: (10, –4.5), directrix: $y = -5.5$ **33.** $y + 4 = \frac{1}{20}(x - 5)^2$
35. $y + 1 = \frac{1}{24}(x - 4)^2$ **37.** $y = \frac{1}{16}(x - 2)^2$
39. 1 **41.** $y = 0.07x^2$ **43. a.** focus: $(0, \frac{1}{4})$, vertex: (0, 0), directrix: $y = -\frac{1}{4}$ **b.** 5 m/s
45. C

Selected Answers

Topic 9

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** parabola **5.** no **7.** yes **9.** $\frac{g-b}{f-a} = -\frac{h-c}{j-d}$; If this equation is true, then the diagonals are perpendicular. If a parallelogram has perpendicular diagonals, it is a rhombus. **11.** $M(-a, b)$ **13.** Calculate the slope of all four sides and the length of all four sides. If one set of opposite sides is parallel but not congruent and the other set of opposite sides is congruent, then the figure is an isosceles trapezoid.
15. $x^2 + y^2 = 81$
17. $(x + 5)^2 + (y + 8)^2 = 13$ **19.** no
21. No; any point that is equidistant from the two given points could be the center of the circle. There are infinitely many such points, so there are infinitely many possible circles.
23. $y + 3.5 = \frac{1}{2}(x + 1)^2$
25. $y - 15 = \frac{1}{100}x^2$

Selected Answers

Topic 10

Lesson 10-1

1. Arc length is a fraction of the circumference proportional to the central angle. Sector area is a fraction of the circle area proportional to the central angle. **3.** A segment of a circle has as one of its boundaries a segment connecting two points on a circle. **5.** measure: 134°; length: $\frac{134}{45}\pi$ **7.** $\frac{\pi}{2}$ radians **9.** $9\pi - 18$ **11.** Steve subtracted the triangle area from the arc length, not the sector area. **13.** 24.3%; Each central angle is 72°, so the area of each triangle is about 11.89.

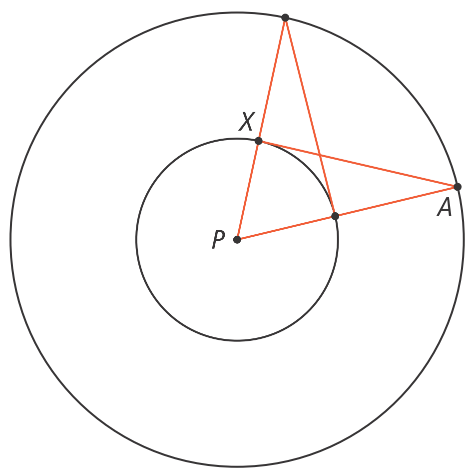
$\frac{\pi(5)^2 - 5(11.89)}{\pi(5)^2} \times 100 \approx 24.3$

15. $\frac{m}{4}$; $\frac{m}{360}\pi r^2 = \frac{x}{360}\pi(2r)^2$, so $\frac{m}{360}\pi r^2 = \frac{x}{360}4\pi r^2$, or $m = 4x$.
17. 50° **19.** 235° **21.** $\frac{415}{4}\pi$ **23.** 115.2
25. 4.8 **27.** $x = 22.25$, $y = 29.31$
29. 933 in.^{2}; Using the Pythagorean Theorem, the height of the triangles associated with the segments is about 11.1 in. Using trigonometry, the angle of the sector is 119.5°. So, the area of each segment is about 293.8 in.^{2}.
$\pi(22)^2 - 2(293.8) \approx 933$. **31.** C

Lesson 10-2

1. A tangent line is perpendicular to the radius of the circle at the point of tangency. **3.** Yes; there is a radius to every point on the circle, and there is a perpendicular to every radius. **5.** no
7. 59° **9.** 17 **11.** Andrew did not apply Theorem 10-2 correctly. By Theorem 10-2, $DE = DH$ and $EF = FG$. So, $DE = 14$.
13. Given: $\overline{QR} \perp n$ **Prove:** n is tangent to $\odot Q$ at R. **Proof:** Let S be any other point on n. Since $\overline{QR} \perp n$, $\triangle QRS$ is a right triangle. Since $\overline{QS}$ is the side opposite the right angle, it is the hypotenuse. Therefore, $QS > QR$. Since $\overline{QR}$ is the radius of the circle and all radii of the same circle are congruent, S lies outside the circle. Therefore, R is the only point of intersection of $\odot Q$ and line n, which means that n is tangent to $\odot Q$ at R.
15. 2x; By Theorem 10-2, $AC = FC$, $AB = BD$, and $DE = EF$. So, $BD + BC = CA$, and $DE + EC = FC$. **17.** 32° **19.** 34

21.



23. 6.1 in. **25. a.** The existing tracks are tangent to the curve, so the angle at P is 74°. Also, P is on the angle bisector of the angle formed by the existing tracks. If x is the distance from where the existing tracks intersect to P, then $\cos 37° = \frac{450}{x}$, so $x \approx 563.5$ m. They should follow the angle bisector from the intersection of the existing track 563.5 m to find P. **b.** about 8.4 km; The length of track from the curve to the intersection of the existing tracks is about 339 m. The length of the curved track is about 581 m. $5 + 3.5 - 2(0.339) + 0.581 \approx 8.4$. **27.** C

Selected Answers

Topic 10

Lesson 10-3

1. Chords in a circle are congruent if their central angles are congruent, and if the central angles are congruent, the chords are congruent. Congruent chords intercept congruent arcs, and congruent arcs are intercepted by congruent chords in a circle. **3.** A chord is a segment whose endpoints lie on a circle. The endpoints of a diameter lie on a circle. Therefore, a diameter is a chord. **5.** 4 **7.** 2 **9.** 4
11. approximately 3.32 ft

13.

Statement	Reason
1) $\angle AEB \cong \angle CED$	1) Given
2) $\overline{EA} \cong \overline{EB} \cong \overline{EC} \cong \overline{ED}$	2) All radii of a circle are congruent.
3) $\triangle AEB \cong \triangle CED$	3) SAS
4) $\overline{AB} \cong \overline{CD}$	4) CPCTC

15.

Statement	Reason
1) $\overline{LN}$ is a diameter of $\odot Q$, $\overline{LN} \perp \overline{KM}$.	1) Given
2) $\angle QPK$ and $\angle QPM$ are right angles.	2) Def. of perpendicular
3) $\overline{QK} \cong \overline{QM}$	3) All radii of a circle are congruent.
4) $\overline{QP} \cong \overline{QP}$	4) Reflexive Property
5) $\triangle QPK \cong \triangle QPM$	5) HL
6) $\overline{KP} \cong \overline{MP}$	6) CPCTC

17. Because the two triangles are similar, the central angles are corresponding angles of similar triangles and therefore congruent. The central angles therefore intercept arcs of equal measure. **19.** 90° **21.** $\overline{PR}$
23. 30 **25.** ≈30.59 cm **27.** 6 **29.** 10 ft
31. a. No; the diameter of the top half is approximately 6.93 in.
b. No; the diameter of the top third is approximately 5.96 in. **33.** B

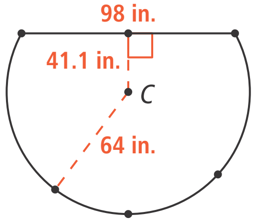
Lesson 10-4

1. The measure of an inscribed angle is one-half the measure of the intercepted arc. **3.** No; the measure of an inscribed angle Is always one-half the measure of the intercepted arc. Since the measure of any arc is positive, half the measure of the arc is less than the measure of the arc. **5.** 98° **7.** 83°
9. 144° **11.** 53° **13.** 33° **15.** 90°
17. $(360 - x)°$; Since chords $\overline{AB}$ and $\overline{CD}$ are congruent, $\widehat{AB} \cong \widehat{CD}$. So, $\widehat{ABC} \cong \widehat{BCD}$, and $m\widehat{BCD} = x°$. Therefore $m\widehat{DAB} = (360 - x)°$. **19.** Draw $\overrightarrow{SC}$ and label the point of intersection A with $\odot C$. Using the Inscribed Angles Theorem, Case 1, $m\angle ASR = \frac{1}{2}m\widehat{AR}$ and $m\angle AST = \frac{1}{2}m\widehat{AT}$. By the Angle Addition Postulate, $m\angle RST = m\angle ASR + m\angle AST$, so using substitution and the Distributive Property, $m\angle RST = \frac{1}{2}\left(m\widehat{AR} + m\widehat{AT}\right)$. Adding the measures of the adjacent arcs, $m\angle RST = \frac{1}{2}m\widehat{RT}$. **21.** No; no; regardless of where Point B is located, $\angle ABC$ will always intercept a semicircle, which is an 180° arc. By the Inscribed Angles Theorem, $m\angle ABC$ will always be 90°. **23.** 250° **25.** 112° **27.** 23°
29. 69° **31.** 63° **33.** Draw $\overrightarrow{SC}$ and label

Selected Answers

Topic 10

the point of intersection Z with $\odot C$. Using the Inscribed Angles Theorem, Case 1, $m\angle RSZ = \frac{1}{2}m\widehat{RZ}$ and $m\angle ZST = \frac{1}{2}m\widehat{TZ}$. By the Angle Addition Postulate, $m\angle RSZ = m\angle TSZ + m\angle RST$, or $m\angle RSZ - m\angle TSZ = m\angle RST$. Using substitution and the Distributive Property, $\frac{1}{2}(m\widehat{RZ} - m\widehat{TZ}) = m\angle RST$. Subtracting the measures of the arcs, $m\angle RST = \frac{1}{2}m\widehat{RT}$. **35.** 67.5° **37.** The center of the circle is about 41.1 in. from the center of the screen, and the radius of the circle is about 64 in. So, to have the same viewing angle, sit at a point on the circle with radius 64 in. that is centered 41.1 in. from the center of the screen. Malcom can sit on any chair on the circle.



39. A

Lesson 10-5

1. When secants intersect, the products of the distances from the point of intersection to each point of intersection with the circle are equal. When secants intersect inside a circle, the angle measures are half the sum of the intercepted arcs. When secants intersect outside a circle, the angle measures are half the difference of the intercepted arcs. **3.** Secants and tangents both intersect a circle, but secants intersect the circle at two points and tangents intersect the circle at one point. **5.** 73° **7.** 10 **9.** 6 **11.** Given: $\odot X$ with secants $\overline{AD}$ and $\overline{AE}$

Prove: $(AD)(AB) = (AC)(AE)$

Statement	Reason
1) $\odot X$ with secants $\overline{AD}$ and $\overline{AE}$	1) Given
2) Draw $\overline{BE}$ and $\overline{CD}$.	2) Two points define a line segment.
3) $\angle BEC \cong \angle CDB$	3) Inscribed $\angle$s intercepting the same arc are $\cong$.
4) $\angle DAE \cong \angle DAE$	4) Reflexive Prop. of $\cong$
5) $\triangle ACD \sim \triangle ABE$	5) AA $\sim$
6) $\frac{AD}{AE} = \frac{AC}{AB}$	6) Ratios of corresp. sides of similar triangles are =.
7) $(AD)(AB) = (AC)(AE)$	7) Mult. Prop. of =

13. ≈19.6 units^2 **15.** Case 1 is when two secants intersect outside of the circle. Case 2 is when a secant and a tangent intersect outside of a circle. Case 3 is when two tangents intersect outside of the circle. **17.** 76°
19.Given: $\odot Q$, tangents $\overrightarrow{AB}$ and $\overrightarrow{AC}$, point D on major arc $\widehat{BDC}$ **Prove:** $m\angle BAC = \frac{1}{2}m\widehat{BDC} - m\widehat{BC}$ **Proof:** Draw chord $\overline{BC}$. Mark point E on $\overrightarrow{AC}$ on the other side of C from A. By Theorem 10-9, $m\angle BCE = \frac{1}{2}m\widehat{BDC}$ and $m\angle ABC = \frac{1}{2}m\widehat{BC}$. From the Triangle Exterior Angle Theorem, $m\angle BCE = m\angle ABC + m\angle BAC$, or $m\angle BAC = m\angle BCE - m\angle ABC$. By substitution and the Distributive Property, $m\angle BAC = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$.
21. 9 **23. Given:** $\odot T$, secants $\overline{JK}$

Selected Answers

Topic 10

and $\overline{LM}$ intersecting at point N
Prove: $(JN)(KN) = (LN)(MN)$ Draw $\overline{JM}$ and $\overline{LK}$. $\angle MJK$ and $\angle KLM$ are inscribed angles that intercept the same arc, so $\angle MJK \cong \angle KLM$. $\angle JNM$ and $\angle LNK$ are vertical angles, so $\angle JNM \cong \angle LNK$. By AA Similarity, $\triangle JNM \sim \triangle LNK$. Since ratios of corresponding sides of similar triangles are equal, $\frac{JN}{LN} = \frac{MN}{KN}$. Therefore, by the Multiplication Property of Equality, $(JN)(KN) = (LN)(MN)$.
25. a. $\approx 45°$ **b.** ≈ 4.8 ft **27.** 140°; Since there are 9 arcs of measure $x°$ around the circle, $x = 40$. $m\angle 1 = \frac{1}{2}(2(40) + 5(40)) = 140°$ **29.** B

Topic Review

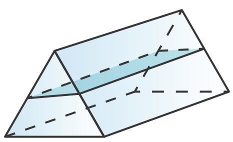
1. Check students' work. See *Teacher's Edition* for details. **3.** inscribed angle **5.** intercepted arc **7.** $\frac{412}{45}\pi$ **9.** $\frac{2,107}{90}\pi$ **11.** $2\sqrt{30}$ **13.** No; the tangents are both perpendicular to the diameter, so they are parallel to each other and never intersect. **15.** 2 **17.** 88° **19.** 89.4 cm

Selected Answers

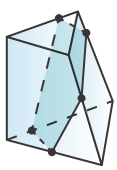
Topic 11

Lesson 11-1

1. The face of a polyhedron is a polygon. For a polyhedron, the sum of the number of faces and vertices is always two more than the number of edges. The cross section of a polyhedron intersected by a plane is polygon. A polygon can be rotated about an axis of rotation to form a three-dimensional figure. **3.** No; even though the numbers satisfy Euler's Formula, three faces is not enough make a polyhedron. **5.** 12 **7.** 64
9. double cone **11.** Philip misstated Euler's Formula. The formula should be $F + V = E + 2$. The polyhedron has 30 vertices.
13. yes



15. a. Yes; **b.** 5

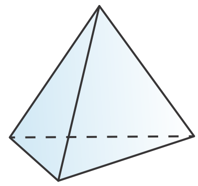


17. 20 **19.** 8 **21.** a square
23. a cylinder **25.** a sphere
27. 12 faces, 14 vertices, 24 edges

29.

Polyhedron	Faces (F)	Vertices (V)	Edges (E)
regular dodecahedron	12	20	30
heptagonal pyramid	8	9	15
octahedron	8	6	12
rhombohedron	6	8	12

31.



Part A Choose to make each face a triangle because triangles have the fewest number of sides and vertices of any polygon. **Part B** If there were fewer than 4 faces, the figure would not be three-dimensional. If there were fewer than 4 vertices and 6 edges, the polyhedron would not satisfy Euler's Formula. $4 + 4 = 6 + 2$

Lesson 11-2

1. The volume of both a prism and a cylinder is the product of the area of its horizontal cross section and its height.
3. Both an oblique prism and an oblique cylinder are slanted; their sides are not at right angles with their bases. The base of an oblique cylinder must be a circle, while the base of an oblique prism can be any polygon. **5.** No; a dent in the can changes the area at that height, so Cavalieri's Principle does not apply. **7.** 62.8 m^3 **9.** 960 cm^3
11. 552.9 in.^{3} **13.** Although the bottles are the same height, they do not have the same cross-sectional area at every height. The cross-sectional area of the square prism is 16 in.^{2}, and the cross-sectional area of the cylinder is 4π in.^{2}, or about 12.56 in.^{2}. **15.** No; rotation about line m produces a cylinder with a diameter equal to the length of the rectangle. Rotation about line n produces a cylinder with a diameter equal to two times the length of the rectangle. **17.** The vases have the same

Selected Answers

Topic 11

height and the same cross-sectional area at every height. By Cavalieri's Principle, the volumes are equal. **19.** 945 g **21.** The trapezoidal ditch holds the greater volume of water. The volume of the trapezoidal ditch is $\frac{1}{2}(40 + 60)(30)(12) = 18{,}000$ in.^{3}, while the volume of the half cylinder ditch is $\frac{1}{2}\pi(30)^2(12) \approx 16{,}965$ in.^{3}. **23.** No; the inner radius of the pool is 3 ft 8 in., or about 3.67 ft. The pool holds $\pi(3.67)^2(3) \approx 126.9$ ft^3 of water, or about $126.9(7.48) \approx 949$ gal of water. It will take Ines about $949 \div 24 \approx 39.5$ min to fill the pool. **25. a.** 4.5 cm **b.** 6 cm **27. Part A** 25.6 kg; no **Part B** 9 short candles **Part C** any combination of 10 candles that includes no more than 8 short candles

Lesson 11-3

1. The formulas for cones and pyramids are both one-third times the volume of a cylinder or pyramid with the same base area and height. **3.** $\frac{h}{3}$ **5.** 126 cm^3 **7.** 69.9 in.^{3} **9.** 3,312 ft^3 **11.** 9 in. **13.** Jacob squared π instead of squaring the radius. He also used the wrong value for the height. **15.** $V = \frac{1}{3}\pi r^3$ **17.** No; the two solids have equal areas at their bases but not at all heights. **19.** 53.3 units^3 **21.** 1,077.6 units^3 **23.** 17.8 units^3 **25.** 26 pieces **27.** 43 min **29.** height; area; volume **31. Part A** No; the volume of Design B is about 14.1 in.^{3}, so it holds about 7.7 fl oz. **Part B** Yes; the volume of Design A is about 42.4 in.^{3}, so it holds about 23.6 fl oz. **Part C** If the diameter is increased to 4 in., the volume would be about 25.1 in.^{3}, so it would hold about 14 fl oz.

Lesson 11-4

1. The volume of a hemisphere of given radius is the volume of a cylinder with same radius and height minus the volume of a cone of the same radius and height. **3.** A great circle is the intersection of a plane containing the center of a sphere and the sphere. The plane divides the sphere into two hemispheres. **5.** 1,017.9 **7.** 523.6 **9.** 33.5 cm^3 **11.** A hemisphere has the same cross-sectional area at a given height above the base as a cylinder with the same height and radius minus a cone with the same height and radius. **13.** $V = \frac{1}{3} \bullet r \bullet \text{S.A.}$ or $V = \frac{1}{(6\,\pi)} \bullet \sqrt{(\pi \bullet (\text{S.A.})^3)}$ **15.** The surface area is one-fourth of the surface area of a whole sphere, $\text{S.A.} = \frac{1}{4} \bullet 4\pi(2)^2 = 4\pi$. The volume is one-fourth of the volume of a whole sphere, $V = \frac{1}{4} \bullet \frac{4}{3}\pi(2)^3 = \frac{8}{3}\pi$. **17.** πr^2 **19.** 37.7 units^2 **21.** 51.7 cm^2 **23.** 9,202.8 units^3 **25.** 3,619.1 ft^3 **27.** 994.8 units^3 **29.** 47 pumps **31.** 11.82 cm **33. i.** A **ii.** D **iii.** C **iv.** B **35. Part A** about 3.26 m^3 **Part B** diameter about 2.06 m, length about 8.22 m.

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** Cavalieri's Principle **5.** 6 **7.** cylinder **9.** 49,480.1 mm^3 **11.** volume of the prism **13.** 112 in.^{3} **15.** $\frac{2}{3}t^3$ **17.** 7.2 in.^{3} **19.** $16r^3$ cm^3

Selected Answers

Topic 12

Lesson 12-1

1. Knowing whether events are mutually exclusive or independent can help you choose appropriate formulas for calculating probabilities. If events A and B are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$. If A and B are independent events, then $P(A \text{ and } B) = P(A) \bullet P(B)$. **3.** Because Deshawn is likely to talk to his friends while playing basketball, the two events are not mutually exclusive. So, he cannot find the probability of one or the other happening by adding them.
5. a. 0.25, 25%, or $\frac{1}{4}$ **b.** 0.75, 75%, or $\frac{3}{4}$
7. $\frac{1}{6}$, $0.1\overline{6}$, or $16.\overline{6}\%$ **9.** Answers may vary. Sample: One situation in which every outcome is equally likely and every outcome is mutually exclusive is rolling a standard number cube. If you add the probabilities for getting each side of a number cube, 1, 2, 3, 4, 5, or 6, the sum is $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$. So, if you add all of the probabilities in an experiment where every outcome is both equally likely and mutually exclusive, the sum of the probabilities should be 1. **11.** No; two events A and B are independent if and only if $P(A) \bullet P(B) = P(A \text{ and } B)$ and $(0.10)(0.15) \neq 0.05$. **13.** no **15.** yes **17.** 0.72 or 72% **19.** 0.28 or 28% **21. a.** $\frac{25}{49}$ **b.** independent **23. a.** $\frac{18}{25}$ **b.** $\frac{1}{100}$ **c.** $\frac{1}{2}$ **d.** $\frac{9}{100}$ **25.** 25% **27. Part A** 36%
Part B 64% **Part C** Yes, because even though the chance of rain on any one day is 50% or less, it is more likely than not to rain at some point during her visit.

Lesson 12-2

1. For any two events A and B, find $P(B \mid A)$ and $P(B)$ and compare them. If they are equal, then events A and B are independent; if not, then A and B are dependent events.
3. By definition $P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$. Division by 0 is undefined, so $P(A)$ cannot equal 0. **5.** Taylor divided $P(\text{red and blue})$ by $P(\text{blue})$ and found $P(\text{red} \mid \text{blue})$ instead of $P(\text{blue} \mid \text{red})$. $P(\text{blue} \mid \text{red}) = \frac{0.05}{0.8} = 0.0625$
7. a. $P(B \mid A) = \frac{2}{9}$ **b.** $P(A \mid B) = \frac{1}{4}$
9. No; $P(B \text{ and } T) = \frac{1}{3}$, $P(T) = \frac{1}{2}$, and $P(B) = \frac{1}{2}$. For the events to be independent, $P(B \mid T) = \frac{P(B \text{ and } T)}{P(T)} = P(B)$ has to be true. But $\frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3} \neq \frac{1}{2}$, so the events are not independent.
11. The events are not independent, so $P(\text{red second} \mid \text{red first})$ is not the same as $P(\text{red first})$. $P(\text{red second and red first}) = P(\text{red first}) \bullet P(\text{red second} \mid \text{red first}) = \frac{3}{10} \bullet \frac{2}{9} \approx 0.067$.
13. a. $\frac{2}{9}$; $P(\text{left crate} \mid \text{vegetable}) = \frac{P(\text{left crate and vegetable})}{P(\text{vegetable})} = \frac{\frac{2}{20}}{\frac{9}{20}} = \frac{2}{9}$
b. $\frac{8}{13}$; $P(\text{left crate} \mid \text{vegetable}) = \frac{P(\text{left crate and vegetable})}{P(\text{vegetable})} = \frac{\frac{8}{20}}{\frac{13}{20}} = \frac{8}{13}$
15. about 0.52 or 52% **17.** 0.45 or 45% **19.** 0.08 or 8% **21.** dependent **23.** 0.225 or 22.5% **25.** $\frac{1}{12}$ or about 0.08 or 8% **27.** A, D

Selected Answers

Topic 12

29. Part A

Student Exercise Survey

	Exercises Daily	Does Not Exercise Daily	Total
Male	28	22	50
Female	32	18	50
Total	60	40	100

Part B $P(\text{female} \mid \text{exercise}) = \frac{8}{15} \approx 0.53$; $P(\text{male} \mid \text{exercise}) = \frac{7}{15} \approx 0.47$
Part C Answers may vary. Sample: Of the students surveyed, 64% of females and 56% of males exercise daily. This suggests that females are somewhat more likely to exercise regularly than males.

Lesson 12-3

1. They provide formulas for finding the total number of outcomes needed to compute probabilities. **3.** There are more permutations of *n* items taken *r* at a time than there are combinations because the order of the items distinguishes between multiple permutations that contain the same items. **5.** No; the compound fraction is simplified incorrectly. 2! should appear in the numerator instead of the denominator, and the correct answer is $\frac{1}{10}$. **7.** combination
9. 1320 **11.** 70 **13.** $\frac{28}{91}$ **15.** $\frac{15}{91}$
17. 24,360; the arrangements are permutations because locks are specific, and if you enter the numbers in the wrong order, locks will not open.
19. The order is important in this problem. The number of ways to select 1 and 6 to form 16 is given by $_6P_2$, not $_6C_2$; $P(16) = \frac{1}{_6P_2} = \frac{1}{30}$.
21. 12; because the people are sitting around a circular table, the first position is determined and the order of the remaining 11 people is 11!.
23. permutation; 3,628,800
25. permutation; 40,320
27. combination(s); 630
29. a. Use a permutation because order matters. $P(DEB) = \frac{1}{_{10}P_3} = \frac{1}{720}$
b. Use combinations because order does not matter.
$P(\text{all vowels}) = \frac{_3C_3}{_{10}C_3} = \frac{1}{120}$
31. $\frac{1}{114}$; there are $_5C_3$ ways to select the tickets he wants and $_{20}C_3$ ways to select 3 tickets. So P(3 winning baseball cap tickets) $= \frac{_5C_3}{_{20}C_3} = \frac{10}{1{,}140} = \frac{1}{114}$.
33. a. $\frac{_6P_6}{_{20}P_6} = \frac{1}{38{,}760}$ **b.** $\frac{1}{_{20}P_6} = \frac{1}{27{,}907{,}200}$ **35.** D

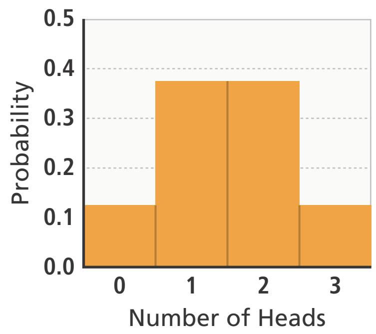
Lesson 12-4

1. A probability distribution for an experiment shows how probabilities are assigned to each outcome of a sample space for the experiment. It can show whether the probability distribution is a uniform probability distribution or not, and if not, which outcomes are more likely or less likely. **3.** Rochelle rounded each probability to the nearest tenth. The sum of the probabilities should be 1, not 1.2. A correct distribution is *P* such that $P(A) = 0.25$, $P(B) = 0.25$, $P(C) = 0.25$, and $P(D) = 0.25$.

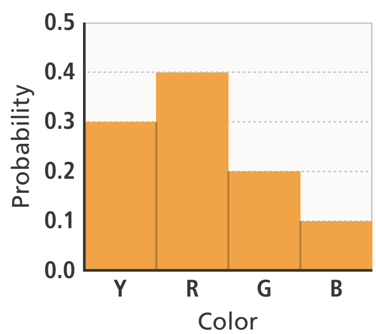
Selected Answers

Topic 12

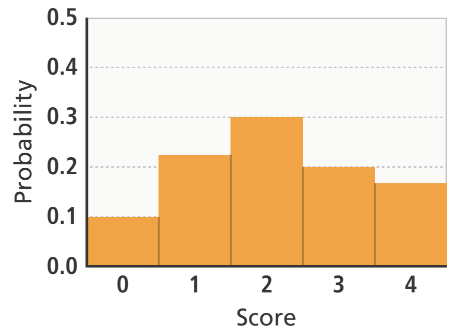
5.



7. 13% **9.** 15% **11.** When you flip a fair coin, the probability of heads and the probability of tails are each $\frac{1}{2}$.
13. No; the results of the two selections are not independent. The teacher will not choose the same student twice, therefore the second selection is being made from fewer students, so the probability of success is different.
15. a. Sample: There are 6 ways to select 2 green marbles in 4 trials: GGRR, GRGR, GRRG, RGGR, RGRG, RRGG. That is, there are ${}_4C_2 = 6$ ways to choose 2 things out of 4. **b.** ${}_5C_3 = {}_5C_2$; choosing 3 out of 5 items is the same as not choosing those 3 items from the 5 items, or choosing 2 out of the remaining 5 items.
17.

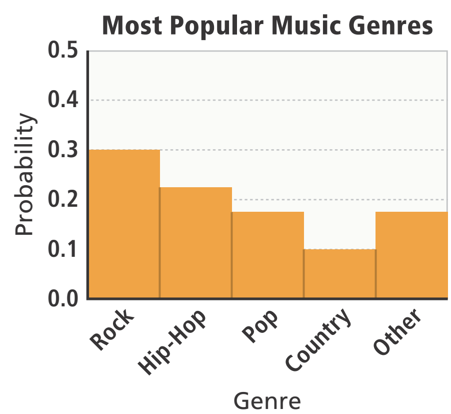


19.



21. No; the results of the two selections are not independent. Since the first card is not returned, the sample space is smaller for the next selection, so the probability changes. **23.** 5.6%
25. 28.2% **27. a.** Let P be the function, defined on the set {Rock, Hip-Hop, Pop, Country, Other} such that $P(\text{Rock}) = 30\%$, $P(\text{Hip-Hop}) = 22\%$, $P(\text{Pop}) = 19\%$, $P(\text{Country}) = 10\%$, $P(\text{Other}) = 19\%$.

b.



c. rock **29.** 1.7% **31.** D

Lesson 12-5

1. Expected value provides a way to create a mean value for a variable that follows a probability distribution.
3. Yes; the carnival will lose an average of \$1.12 – \$1 = \$0.12 each time the game is played. The expected payout must be less than \$1 for the carnival to earn money.

Selected Answers

Topic 12

5. when there are equal numbers of each kind of item **7.** 7
9. a. $960 **b.** No; expected value tells you the average cost if you were booking many such flights.
11. The student incorrectly calculated the probabilities for each outcome. The area of the entire dartboard is 49π and the area of the inner circle is 4π. So, the area of the outer region is $49\pi - 4\pi = 45\pi$. Therefore, the probability that a dart lands in the inner circle is $\frac{4\pi}{49\pi} = \frac{4}{49}$, the probability that it lands in the outer region is $\frac{45\pi}{49\pi} = \frac{45}{49}$, and the expected value is $\frac{4}{49}(4) + \frac{45}{49}(1) = \frac{61}{49} \approx 1.24$. **13. a.** The cost of the policy with no deductible is $600, so the policy with the $1,000 deductible costs less. **b.** Despite the fact that the policy with a deductible is less expensive, the consumer might choose the policy with no deductible to avoid an unexpected expense of $1,000 sometime during the year.
15. $0.04 per dozen **17.** $76.89
19. 328.5 days **21.** In the next year, if the company makes no changes, about 0.01 • 30,000 = 300 panels will be defective, incurring a cost of 300 • $600 = $180,000. If they make the manufacturing changes, the expected total cost is $200,000 + 0.002 • 30,000 • $600 = $236,000. So, for the next year, it makes sense to not improve the process. But the company also predicts that sales will increase by 5,000 per year. Yearly costs of defective panels:

Year	Sales	Cost (in $1000s) With No Change	Cost (in $1000s) With Improved Process
1	30,000	$180	$236
2	35,000	$210	$242
3	40,000	$240	$248
4	45,000	$270	$254
5	50,000	$300	$260
6	55,000	$330	$266
7	60,000	$360	$272
8	65,000	$390	$278
9	70,000	$420	$284
10	75,000	$450	$290

For 3 years it is less costly to make no change. After 3 years, improving the process becomes a better option.
23. No warranty, because the expected cost for 3 years is less with no extended warranty than with a 2-year or 3-year warranty. Expected costs for 3 years: No extended warranty:
$599 + $278(0.05) + $278(0.08) = $635.14 2-year extended warranty:
$599 + $55 + $278(0.08) = $676.24
3-year extended warranty:
$599 + $80 = $679.00 **25.** 14; 1
27. Part A Each side of the center square should be $\frac{1}{3}$ the side length of the large square. The ratio of the areas of the squares is 1:9, so the two regions have areas in the ratio 1:8. **Part B** 3 points; Let *x* be the point value in the outer region. The probability of a chip landing in the center square is $\frac{1}{9}$ while the probability of landing in the outer region is $\frac{8}{9}$. Since a chip in the center square is worth 20 points, the expected value is $\frac{1}{9}(20) + \frac{8}{9}(x)$. For an expected value close to 5, solve this equation.

Selected Answers

Topic 12

$$\frac{1}{9}(20) + \frac{8}{9}(x) = 5$$
$$\frac{20}{9} + \frac{8x}{9} = 5$$
$$20 + 8x = 45$$
$$8x = 25$$
$$x = 3.125$$

So, making the outer region worth 3 points solves the problem.
Part C The expected score is increased but not necessarily doubled. The amount of increase in the expected score depends on the point value of landing in each area.

Lesson 12-6

1. The probability of an outcome multiplied by its value can help you decide whether a game is fair, find the costs and benefits for a risky business venture, and make rational decisions based on expected values and returns rather than guesses. **3.** No; how the numbers are assigned or what meaning is attributed to them is a separate factor that determines whether a game is fair. **5.** Answers may vary. Sample: Inspecting the graph of a probability distribution or a tree diagram makes it easier to see how likely or unlikely certain outcomes are, and inform your decisions. **7.** student 9, then student 4 **9.** Yes; they each have a $\frac{1}{6}$ chance of winning. **11.** Answers may vary. Sample: **a.** Roll your dice. If the product is even you get one point, if the product is odd you lose two points. **b.** Roll your dice. If the sum is prime number you get two points, if the sum is composite you lose one point. **13.** The error is in the exponents for the probability terms. The correct formula is $(12)(0.80)^{11}(0.20)^{1} + (1)(0.80)^{12}(0.20)^{0}$. The correct probability is 27%. **15.** Fair; even numbers have the same number of chances, {2, 4, 6}, as odd numbers {1, 3, 5}. **17.** \$33,875 **19. a.** about 0.00025, or 0.025% **b.** No; the probability is 0.025% that 2 or more people selected at random from a group of 5 people with an AGI between \$50,000 and \$75,000 will be audited. However, 2 out of 5 people in that tax bracket whose returns were prepared by ABC Tax Guys were audited, which is 40%. So I would not recommend ABC Tax Guys to a friend. **21. a.** 40% (rounded to the nearest whole percent) **b.** Based on the test of 50 new components, the experimental probability for failure is 2%. However, 50 is a relatively small sample, so I think that it is not reasonable to conclude that the new components have a lower failure rate than 4%. **c.** Further testing of the new parts is recommended, since 50 is a relatively small sample. **23.** E

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** mutually exclusive events **5.** complement **7.** not mutually exclusive **9.** not mutually exclusive **11.** 0.58 **13.** 0.40 **15.** dependent events **17.** 0.0075 or 0.75% **19.** permutation; 720 **21.** Permutations and combinations are ways to count an arrangement of items. The arrangements cannot occur in fractional ways. **23.** 6.73% **25.** 0.30% **27.** 423.75

Selected Answers

Topic 12

29. The player's calculation is correct: 3 points times 0% is 0. However, the player is basing the expected value on very few trials, so the value is likely to be inaccurate. If the player takes 20, 50, or 100 shots and bases her expected value calculation on the number of those shots that are successful, her estimate is likely to be better.
31. $x = 18.75$